MA 1128: Lecture 05 – 2/2/2018

Inequalities
And Absolute Values
Inequalities

An *inequality* looks like an equation, except it has a $<$, $>$, $\leq$, $\geq$, or $\neq$ instead of a $=$.

The symbols $<$ and $>$ read

\[
2 < 4 \quad \text{2 is less than} \quad 4 \\
4 > 2 \quad \text{4 is greater than} \quad 2 
\]

If you think of the $<$ and $>$ as arrows, note that they always point to the smaller number.

The $\leq$ and $\geq$ are combination symbols, and they can be satisfied in two ways.

For example,

\[
2 \leq 4 \quad \text{2 is less than or equal to} \quad 4 \\
\]

And also

\[
2 \leq 2 \quad \text{2 is less than or equal to} \quad 2 
\]
Inequalities (cont.)

The $\neq$ means *not* equal.
For example,

$$2 \neq 4 \quad \text{2 is not equal to 4}$$

The most important thing to remember about inequalities is the following.
If you multiply (or divide) both sides of an inequality by a negative number, then the direction of the inequality reverses direction.
Example

$$-4 < 3$$

$$(-3)(-4) > (-3)(3)$$

$$12 > -9$$
Practice Problems

For each of the following, state whether the given inequality is true or false.
1. $3 < 5$
2. $-3 \geq -5$
3. $2 \geq 2$
4. $2 \geq 5$
5. $4 \neq 4$
6. $-3 < -5$
7. $-4 < -2$
8. $(-2)(-4) > (-2)(-2)$
9. $(-2) \cdot x > (-2) \cdot y$, if you know that $x < y$.

Click for answers
1) T; 2) T; 3) T; 4) F; 5) F; 6) F; 7) T; 8) T; 9) T.
Solving Linear Inequalities

We can solve linear inequalities exactly the same way we solve linear equations… **EXCEPT we reverse the inequality whenever we multiply by negatives.**

For example.

\[-2x + 7 \geq 9\]

\[-2x + 7 - 7 \geq 9 - 7\]

\[-2x \geq 2\]

\[-\frac{2x}{-2} \leq \frac{2}{-2}\]

\[x \leq -1\]

This last inequality is as simple as possible, and the solutions are easy to see. Any \( x \) that is less than or equal to \(-1\) is a solution.
Practice Problems

Just leave your solutions with fractions (e.g., $x > 2/3$)

1. Solve the inequality $7x - 3 > 2$
2. Solve the inequality $4 - 3x < 7 + 2x + 4$
3. Solve the inequality $4[ x - (3x - 2) ] \geq 3(x + 2) - 6$. (Hint: I would simplify the rounded parentheses first, then the square ones, before solving the inequality.)

Click for answers

1) $x > 5/7$; 2) $x > -7/5$; 3) $x \leq 8/11$. 
Absolute Values

Absolute values are indicated with vertical bars. The *absolute value* of a number is the distance it is from 0.

Example.

\[ |3| = 3 \]
\[ |-7| = 7 \]

Roughly, the \( | | \) tell us to take whatever is inside, and make it positive. Note that if the number inside is positive (or zero), we do nothing. And if the number inside is negative, we *change* the sign (to make it positive).

**Remember:** What we do with an absolute value depends on whether what’s inside is positive or negative.
Equations with Absolute Values

Consider the equation

\[ |x| = 3 \]

This has two solutions, \( x = 3, -3 \).

On the other hand, the equation

\[ |x| = -4 \]

has no solutions, since \( |x| \) is positive (or zero) and cannot possibly be equal to the negative number, \(-4\).

Whenever we want to solve an equation with absolute values, we will always break it into cases, and solve several different equations \textit{without} absolute values.
Example

$|x - 3| = 7$

This equation is true if what’s inside the absolute value is $7$ or $-7$. In other words, if

$x - 3 = 7$

OR

$x - 3 = -7$

We can solve each of these as we would any linear equation.

$x - 3 + 3 = 7 + 3$

$x = 10$

and

$x - 3 + 3 = -7 + 3$

$x = -4$

The solutions are $x = 10, -4$. 

Next Slide
Practice Problems

1. Simplify (write without absolute value symbols) $|3|$.
3. Simplify $|-x|$, if $x$ is a negative number. Hint: Put a negative number in for $x$, and see what you get.
4. Solve $|x + 2| = 5$.
5. Solve $|3x - 2| = 2$.

Click for answers.
1) 3; 2) 4; 3) $-x$; 4) $x = 3, -7$; 5) $x = 4/3, 0$; 6) no solutions.
Example

In the following, it’s easiest to see what to do when the absolute value is on one side by itself.

\[
\frac{5x-3}{2} + 2 = 6\\
\frac{5x-3}{2} + 2 - 2 = 6 - 2\\
\frac{5x-3}{2} = 4
\]

**Case I**

\[
\frac{5x-3}{2} = 4\\
5x - 3 = 8\\
5x = 11\\
x = \frac{11}{5}
\]

**Case II**

\[
\frac{5x-3}{2} = -4\\
5x - 3 = -8\\
5x = -5\\
x = -1
\]

The solutions are \( x = 11/5, -1 \)
Example

There are a lot of ways that absolute values can be in an equation. We will only consider one other case. Two absolute values set equal to each other.

\[ |x - 1| = |2x - 4| \]

The things inside, \( x - 1 \) and \( 2x - 4 \), are made positive by the absolute values. There are four ways the equation can be true: \( x - 1 \) and \( 2x - 4 \) can be positive and positive, positive and negative, negative and positive, and negative and negative.

All of these cases can be described by:

They have the same sign, OR they have opposite signs.
Example (cont.)

Case I, if they have the same signs, then the equation is satisfied if
\[ x - 1 = 2x - 4, \]

and Case II, if they have different signs, then the equation is satisfied if
\[ (x - 1) = -(2x - 4). \]

Note that \(-(x - 1) = -(2x - 4)\) and \-(x - 1) = (2x - 4)\ are covered under Case I and Case II respectively.
Practice Problems

1. Solve Case I in this last example.
2. Solve Case II.
3. What are the solutions to $|x - 1| = |2x - 4|$, the equation of the last example?
4. Solve the equation $|3x - 2| = |2x - 8|$.

Click for answers:
1) $x = 3$; 2) $x = 5/3$; 3) $x = 3, 5/3$ (or $x = 5/3, 3$ it doesn’t matter); 4) $x = -6, 2$.

End