MA 1128: Lecture 08 – 03/02/2018

Linear Equations from Graphs
And Linear Inequalities
Linear Equations from Graphs

Given a line, we would like to be able to come up with an equation for it. I’ll go over two ways of doing this. I don’t care which one you use, but you should be aware of each.

Method I.
I prefer working from the slope-intercept form of a line ($y = mx + b$), since it is the same as the form of a linear function, and we like functions (right?). If you know the slope and a point on the line, you can find the slope-intercept form of the equation as follows.
Example

Suppose the slope of the line is \( m = 3 \), and \((2,1)\) is a point on the line. The fact that \( m = 3 \) immediately tells us that the equation looks like

\[
y = 3x + b.
\]

We just need to find the value for \( b \), which is the \( y \)-intercept. Since \((2,1)\) is a point on the line (i.e., \( x = 2 \) and \( y = 1 \) satisfy the equation), we can substitute these values to find \( b \).

\[
(1) = 3(2) + b, \\
1 = 6 + b, \\
and \ -5 = b.
\]

The equation is, therefore, \( y = 3x - 5 \).
Example

Suppose the slope is \( m = -3/2 \) and \((4, -3)\) is a point on the line.
We know \( y = (-3/2)x + b \),
and therefore, \( -3 = (-3/2)(4) + b \),
and \(-3 = -6 + b\),
and \(3 = b\).

Therefore, the equation is \( y = (-3/2)x + 3 \).

Note that I’m writing fractions the way I am, because it’s easier to do it this way in PowerPoint. On paper, I would write it more like this.

\[
y = -\frac{3}{2}x + 3
\]
Practice Problems

1. Suppose the slope is $m = 2$ and $(4,5)$ is a point on the line. Find the equation.

2. Suppose the slope is $m = \frac{1}{2}$ and $(6,1)$ is a point on the line. Find the equation.

Click for answers:

1) $y = 2x - 3$;

2) $y = (1/2)x - 2$. 

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Method II

Suppose we know the slope of the line is $m = 2$ and $(-3,1)$ is on the line. If a point $(x,y)$ is another point on the line, then we could compute the slope with $m = (y - 1)/(x - (-3)) = (y - 1)/(x + 3)$. Since $(x,y)$ could be any point on the line, we don’t have specific values for them, but we already know the slope is $m = 2$. Therefore, we have the equation $2 = (y - 1)/(x + 3)$.

We can multiply $x + 3$ times both sides to get $2(x + 3) = (y - 1)$.

Or flip things around to get $y - 1 = 2(x + 3)$.

This equation is said to be in point-slope form, since the point $(-3,1)$ and the slope $m = 2$ are readable from the equation.
Point-Slope Form

In general, if $m$ is your slope, and $(x_1, y_1)$ is a point on your line, then

$$y - y_1 = m(x - x_1)$$

is the equation of the line in point-slope form.

For example, if $m = -3/2$ and $(2, -5)$ is on the line, then the equation is

$$y - (-5) = (-3/2)(x - 2).$$

Method II is faster, but I don’t use it very much, because I don’t want to have to remember another form, and I usually want to convert it to slope-intercept form anyway.
Practice Problems

1. Find the equation of the line if $m = -3$ and $(2,1)$ is on the line. Give your answer in slope-intercept form.

Click for answers:
In point-slope form we have $y - 1 = -3(x - 2)$.
This becomes $y - 1 = -3x + 6$,
which becomes $y = -3x + 7$. 
Finding the equation from two points

Sometimes you need to find the equation from two points. For example, suppose \((3,7)\) and \((2,1)\) are on the line. The slope is \(m = (1 - 7)/(2 - 3) = (-6)/(-1) = 6\). Now we can use either Method I or Method II using \(m = 6\) and \((3,7)\). Or just as easily, we can use \(m = 6\) and \((2,1)\), since any point on the line will work fine.
Practice Problems

1. Find the equation of the line (in slope-intercept form) using \( m = 6 \) and (3,7).
2. Find the equation using \( m = 6 \) and (2,1).
3. Were the equations the same?

Click for answers:
In both cases, you should get \( y = 6x - 11 \).
Linear Inequalities

This is a completely new topic. We would like to find the set of solutions to a linear inequality in two variables. (E.g., \( y > -3x + 2 \).)

We will generally indicate the solutions in terms of a graph.

We proceed as follows.

First graph the corresponding linear equation. 
\[
y - \text{int} = 2, \text{ slope } = -3 = -3/1 = \text{ down 3, right 1. } \]

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Example (cont.)

All the points \textit{on} this line make $y$ and $-3x + 2$ \textit{equal} to each other.

\textbf{Basic Principle:} The line divides the plane into two pieces.
All the points on one side of the line satisfy $y < -3x + 2$ (less than).
All the points on the other side of the line satisfy $y > -3x + 2$ (greater than).

The only question is, which side is which?
I think the easiest way is to pick a point on one side of the line and check.
An easy point to check is $(0,0)$, which is below this line (and to the left).
$0 < -3(0) + 2$.
Since $0$ is less than $2$, $(0,0)$ must be on the “$<$“ side of the line.
Example (cont.)

Our inequality is \( y > -3x + 2 \).

Since \((0,0)\) is on the “less than” side, we want the *other* side, the “>” side.
We don’t want “exactly equal” either, so we’ll indicate this with a broken line.
The book uses shading, and that’s pretty standard.
I can’t do that here very easily, so I’ll use arrows.
Example

Consider the inequality \( 3x - 2y \leq -6 \).

First we graph the equation \( 3x - 2y = -6 \). The intercepts are (-2,0) and (0,3), and we want *equals*, so we draw the line *solid*.

We check a point not on the line, say (0,0). We get \( 3(0) - 2(0) \geq -6 \).

This is \( > \), i.e. the wrong side. Shade (or draw arrows) on the other side of the line.
Example

Consider the inequality \( y < \frac{2}{5}x \).

Graph the line (broken): y-intercept (0,0). Up 2, right 5 gives (5,2).

Since (0,0) is on the line, we have to use another point.

Check (1,0): 0 \( < \) \( \frac{2}{5} \)(1). This is <, the correct side.
Practice Problems

7. Graph the inequality \( y < 4x - 2 \).
8. Graph the inequality \( 3x - 4y \geq 12 \).

Click for answers:

1) 

2)