MA 1128: Lecture 11 – 3/23/2018

Factoring
Part I
Factoring Monomials

In Lecture 10, we looked at multiplying polynomials. Today, we’ll start looking at doing the reverse. This is called *factoring*. Consider the product \( 3x^2(6x^2 + 2x - 3) \).

This is a polynomial with one term (a monomial, \( 3x^2 \)) times a polynomial with three terms (\( 6x^2, 2x, \) and \( -3 \)).

\[
= (3x^2)(6x^2) + (3x^2)(2x) + (3x^2)(-3) \\
= 18x^4 + 6x^3 - 9x^2.
\]

When we work with polynomials, there are two ways that we want to see them. Sometimes we want them all multiplied out like \( 18x^4 + 6x^3 - 9x^2 \). Other times we want to see them written as a product of simpler polynomials.

In \( 3x^2(6x^2 + 2x - 3) \), the monomial \( 3x^2 \) will be considered very simple, because it has only one term. The other factor \( 6x^2 + 2x - 3 \) might break down into simpler factors.
Example

If we look at the last polynomial \( 18x^4 + 6x^3 - 9x^2 \), we already have seen that this can be written out as

\[
= (3x^2)(6x^2) + (3x^2)(2x) + (3x^2)(-3).
\]

In particular, \( 3x^2 \) is a factor in every term.
Whenever we have a factor common to every term, we can factor it out.

\[
= 3x^2(6x^2 + 2x - 3).
\]

Multiplying polynomials and factoring them are manifestations of the distributive property.
Example

Consider the polynomial $10x^3 + 6x^2 + 4x$.
We can factor out monomials in stages, if we want.
Since all of the coefficients are even, we can factor out a 2.

$$= 2(5x^3) + 2(3x^2) + 2(2x)$$

$$= 2(5x^3 + 3x^2 + 2x)$$

The 5, 3, and 2 don’t have a common factor,
but we can write $x^3 = (x)(x^2)$, $x^2 = (x)(x)$, and $x = (x)(1)$.

$$= 2( x(5x^2) + x(3x) + x(2) )$$

$$= 2x(5x^2 + 3x + 2).$$

It’s probably just as easy to factor out all at once, as in the next example.
Examples

Consider \(12x^4 - 6x^3 + 18x^2\).
I see that 12, \(-6\), and 18 have a common factor of 6.
The \(x^4\), \(x^3\), and \(x^2\) have a common factor of \(x^2\).
\[= 6x^2(2x^2) + 6x^2(-x) + 6x^2(3)\]
\[= 6x^2(2x^2 - x + 3)\]

Consider \(10x^3 + 5x^2 - x\).
The 10, 5, and \(-1\) have no common factors, but we can factor an \(x\) out.
\[= x(10x^2) + x(5x) + x(-1)\]
\[= x(10x^2 + 5x - 1)\].

**Remember**, if a term gets everything factored out, you’re left with a 1.
Practice Problems

Factor out the biggest monomial that you can.

1. \(6x^3 + 9x^2 + 6x\).
2. \(15x^4 - 5x^3 + 10x^2\).

Answers:
1) \(3x(2x^2 + 3x + 2)\);
2) \(5x^2(3x^2 - x + 2)\)
Factoring Trinomials

Factoring is generally a hit and miss process, but there is one case that can be done systematically. These are the trinomials of the form $x^2 + bx + c$ (the $x^2$-term has coefficient 1). These come from products of the form

$$(x + 3)(x - 2)$$

$$= (x)(x) + (x)(-2) + (3)(x) + (3)(-2)$$

$$= x^2 - 2x + 3x - 6$$

$$= x^2 + x - 6.$$ 

To factor a polynomial in the form $x^2 + bx + c$, we’ll look for two factors of the form $(x + A)(x + B)$. 

Next Slide
With this assumption, we can do some detective work. Look at \((x + 3)(x - 2)\) again.

\[
(x + 3)(x - 2) = (x)(x) + (x)(-2) + (3)(x) + (3)(-2)
\]

\[
= x^2 - 2x + 3x - 6
\]

One term will always be \(x^2\).

There will always be two \(x\)-terms (note that 3 and -2 are coefficients), and one constant term (note that \(3)(-2) = -6\).

\[
= x^2 + x - 6.
\]

Two things have to happen with the constant terms 3 and -2.

3 times -2 equals -6
3 plus -2 equals 1
Example

Consider the trinomial \( x^2 - 3x - 10 \).
To factor into \((x + A)(x + B)\), we need \(AB = -10\).
What pairs of whole numbers multiply together to give \(-10\)?
(1)(-10), (-1)(10), (2)(-5), and (-2)(5). That’s it (the order doesn’t matter).
We also need \(A + B = -3\) (from the \(x\)-term).
Let’s check our four factorings of \(-10\).
(1) + (-10) = -9, no. (-1) + (10) = 9  no. (2) + (-5) = -3  yes! (-2) + (5) = 3  no.
It looks like we want \(A\) and \(B\) to be 2 and -5 (it doesn’t matter which is which).
Factoring: \(x^2 - 3x - 10 = (x + 2)(x - 5)\)
[[ \((x - 5)(x + 2)\) is the same answer. ]]
Practice Problems

1. Multiply out \((x + 2)(x - 5)\). Does this give you \(x^2 - 3x - 10\)?

Answer:
This should work. Note that you can always check a factoring by multiplying it out.
Example

Consider $x^2 - 5x + 6$.
We need $AB = 6$: (1)(6), (-1)(-6), (2)(3), and (-2)(-3).
[[ Don’t forget the negative possibilities!! ]]
We also need $A + B = -5$: (-2) + (-3) = -5 !!
Therefore, $x^2 - 5x + 6 = (x - 2)(x - 3)$.

Consider $x^2 + 11x - 12$.
We need $AB = -12$: (1)(-12), (-1)(12), (2)(-6), (-2)(6), (3)(-4), and (-3)(4).
We need $A + B = 11$: (-1) + (12) = 11 !!
Therefore, $x^2 + 11x - 12 = (x - 1)(x + 12)$.
1. Consider the trinomial $x^2 + 2x - 3$. List out all of the ways you can get $AB = -3$. (there are only two ways).

2. Which, if any, give $A + B = 2$?

3. How does $x^2 + 2x - 3$ factor?

4. Factor $x^2 - 7x + 6$. (there are four ways to get $AB = 6$)

5. Factor $x^2 - 8x + 16$. (there are six ways to get $AB = 16$)

6. Factor $x^2 - x - 12$.

Answers:

1) $(1)(-3)$ and $(-1)(3)$; 2) $(-1) + (3) = 2$; 3) $(x - 1)(x + 3)$;
4) $(x - 1)(x - 6)$; 5) $(x - 4)(x - 4)$; 6) $(x + 3)(x - 4)$
Example

Consider \( x^2 - 10x + 6 \).

We need \( AB = 6 \): \((1)(6), \ (-1)(-6), \ (2)(3), \text{ and } \ (-2)(-3)\).

We need \( A + B = -10 \): \((1) + (6) = 7, \ (-1) + (-6) = -7, \ (2) + (3) = 5, \text{ and } \ (-2) + (-3) = -5\).

None of these work.

This means that \( x^2 - 10x + 6 \) does not factor using whole numbers.

It may factor using messy numbers, but we’ll talk about that later.