MA 1128: Lecture 12 – 3/26/2018

Special Factoring Forms
Solving Polynomial Equations
Special Factoring Forms

There are several special forms that you should memorize. These correspond to products of the form

\[(x + A)(x - A)\]

\[(x + A)(x + A) = (x + A)^2\]

\[(x - A)(x - A) = (x - A)^2\].
Difference of Squares

If we multiply \((x + A)(x - A)\),
where the constant terms are the same except that they have different signs,
the \(x\)-terms cancel out.
For example, \((x + 2)(x - 2)\)
\[= x^2 - 2x + 2x - 4\]
\[= x^2 - 4.\]

Anytime you see a difference of squares,
you should be able to factor it immediately.
Examples

Consider $x^2 - 16$. ($16 = 4^2$)

$= (x + 4)(x - 4)$

[[ or $(x - 4)(x + 4)$. The order doesn’t matter. ]]

Consider $x^2 - 9$. ($9 = 3^2$)

$= (x + 3)(x - 3)$

For $x^2 - 1$. ($1 = 1^2$)

$= (x + 1)(x - 1)$. 
Practice Problems

1. Factor \( x^2 - 9 \).
2. Factor \( x^2 - 25 \).
3. Factor \( x^2 - 49 \).

Answers:
1) \((x + 3)(x - 3)\);
2) \((x + 5)(x - 5)\);
3) \((x + 7)(x - 7)\).
More on difference of squares

You can factor a difference of squares using the technique from the last lecture (Lecture 11).

For example, \( x^2 - 4 = x^2 + 0x - 4 \).

\( AB = -4 \): \( (1)(-4), (-1)(4), (2)(-2) \).

\( A + B = 0 \): \( (2) + (-2) = 0 \) !!

Therefore, \( x^2 - 4 = (x + 2)(x - 2) \).
Sums of Squares Never Factor

If we try to factor $x^2 + 9$ like we did in the last example, we get

$AB = 9$: (1)(9), (-1)(-9), (3)(3), and (-3)(-3).

$A + B = 0$: (1) + (9) = 10. (-1) + (-9) = -10. (3) + (3) = 6. (-3) + (-3) = -6.$

None of these work.

A sum of squares never factors (with real numbers).
Perfect Square Trinomials

If we multiply \((x + A)(x + A)\) or \((x + A)^2\), we get something like

\[ (x + 2)(x + 2) = x^2 + 2x + 2x + 4 = x^2 + 4x + 4. \]

Or \((x - 3)(x - 3)\)

\[ = x^2 -3x -3x + 9. \]

The constant term is *always positive* and the square of \(A\). The \(x\)-term is always \(2Ax\), and it has the same sign as in the factors.
Examples

Factor \( x^2 + 8x + 16 \).
Note that 16 is a square: \( 16 = 4^2 \) and it is also \( 16 = (-4)^2 \).
Also note that \( 2(4) = 8 \) (the coefficient of the \( x \)-term).
This is a perfect square trinomial.
Therefore, \( x^2 + 8x + 16 = (x + 4)(x + 4) = (x + 4)^2 \)

Factor \( x^2 - 20x + 100 \).
100 = 10^2 = (-10)^2, and \( 2(-10) = -20 \).
Therefore, \( x^2 - 20x + 100 = (x - 10)(x - 10) = (x - 10)^2 \).
Be Careful

Always make sure that you have a perfect square trinomial.

Factor \( x^2 + 10x + 16 \).
\[ 16 = 4^2 = (-4)^2, \text{ but } 2(4) \neq 10, \text{ so this is NOT a perfect square trinomial.} \]
It does factor, however, as \((x + 2)(x + 8)\).

Factor \( x^2 + 6x - 9 \).
This is not a perfect square trinomial, because the constant term is negative.
Example

Even if you don’t notice a perfect square trinomial, the old technique still works. For example, \( x^2 - 6x + 9 \).

\( AB = 9 \): (1)(9), (-1)(-9), (3)(3), and (-3)(-3).

\( A + B = -6 \): (-3) + (-3) = -6 !!

Therefore, \( x^2 - 6x + 9 = (x - 3)^2 \)
Practice Problems

1. Factor \( x^2 + 4x + 4 \).
2. Factor \( x^2 - 10x + 25 \).
3. Factor \( x^2 - 10x - 25 \).
4. Factor \( x^2 - 15x - 16 \).
5. Factor \( x^2 - 14x + 49 \).

Answers:
1) \( (x + 2)^2 \); 2) \( (x - 5)^2 \); 3) Does not factor with whole numbers.
4) \( (x + 1)(x - 16) \) – not a perfect square trinomial;
5) \( (x - 7)^2 \).
Solving Polynomial Equations

An equation like $x^2 - 2x + 3 = 0$ can’t be solved like a linear equation. There is a trick we can use, however.

If two numbers multiply together to get zero, then at least one of the numbers must be zero.

$(0)(7) = 0$
$(12)(0) = 0$
$(2)(3) \neq 0$
To solve $x^2 - 2x - 3 = 0$, factor the polynomial.

$(x + 1)(x - 3) = 0$.

The only way $(x + 1)$ times $(x - 3)$ can be zero is if one of these factors is zero. We can figure this out by solving two linear equations. These are $x + 1 = 0$ and $x - 3 = 0$, which are easy to solve, $x = -1, 3$. 

(Cont.)
Example

Consider the equation \( x^2 + 5x = -6 \).

To use what we know, we need a product equaling zero, so move the \(-6\) to the other side to get

\[ x^2 + 5x + 6 = 0. \]

This is always the first thing you want to do.

Now factor.

\[ (x + 2)(x + 3) = 0. \]

Then solve the two equations \( x + 2 = 0 \) and \( x + 3 = 0 \).

This gives \( x = -2, -3 \).
Quadratic Equations

An equation involving only $x^2$-terms, $x$-terms, and constant terms is called a quadratic equation.

If possible, the quickest way to solve a quadratic equation is to get 0 on one side, and then to factor.

For example, $x^2 = -4x - 4$.
This becomes $x^2 + 4x + 4 = 0$ (a perfect square trinomial).
$(x + 2)(x + 2) = 0$.
Therefore, $x = -2, -2$. We’ll usually just write $x = -2$. 
Practice Problems

Solve the following quadratic equations by factoring.
1. Solve \( x^2 - 5x + 6 = 0 \).
2. Solve \( x^2 + 4x - 12 = 0 \).
3. Solve \( x^2 - 6x + 9 = 0 \).
4. Solve \( x^2 - 6x = -8 \).

Answers:
1) \((x - 2)(x - 3) = 0\), so \( x = 2, 3 \);
2) \((x - 2)(x + 6) = 0\), so \( x = 2, -6 \);
3) \((x - 3)(x - 3) = 0\), so \( x = 3 \);
4) \((x - 2)(x - 4) = 0\), so \( x = 2, 4 \).