Objectives: Warm up to viewing objects in 3 dimensions.

For the most part, we will be extending concepts covered in Calculus I and II to three dimensions. Most of these things will be related to higher dimensional versions of derivatives and integrals, and these extensions are generally much less exotic than you might expect. Most of the difficulties will come from the fact that we have 2D eyes.

I’d like to start a little slowly with some visualization exercises and a concept that will keep popping up that I’ll call orientation.

**Some stuff about axes**

Most of the stuff you did in Calculus I and II took place in the $xy$-plane. Much earlier in your life, math took place on the (real) number line. On the real line, 0 was kind of in the middle (whatever that means), positive numbers went to the right, and negative numbers went to the left. Positive to the right and negative to the left was somewhat arbitrary, and having the positives go the left would have worked out just fine. There really isn’t a fundamental difference between the two ways of laying out the real line.

In two dimensions, we have the real plane, $\mathbb{R}^2$. We put coordinates on the plane using two real lines, the $x$-axis and the $y$-axis. We didn’t have to put the two axes at right angles, but that makes things simpler. The negative $x$-axis and the positive $x$-axis need to line up, of course, but we made several arbitrary choices. We put the $x$-axis horizontal, and we put the positive $x$- and $y$-axes to the right and upwards.

**Quiz 01A**

1. I used the word *middle* in describing where 0 is on the real line. One idea of *middle* would mean that there’s the same amount of “stuff” around the middle in all directions. Is that true of 0? Is it true of 5?

2. How about $(0, 0)$ in the $xy$-plane? The point $(-1, 3)$?

   Once we put a coordinate system on a line or a plane, we can start to tell points apart. But without a coordinate system, all the points are basically the same. There are infinitely many different points, but one point is not any more in the middle than any other. We can center our coordinate system with 0 or $(0, 0)$ anywhere we want. In some sense, we study coordinate systems on lines and planes, rather than lines and planes. It’s not really a huge deal, of course, but it’s something to consider.

In Calculus III, we’ll be working with $xyz$-space. Our basic conception of space is that it’s 3D or three dimensional. Planes are 2D, and lines are 1D.

I will have a peculiar way of categorizing objects as 2D, or 3D, etc. In particular, an object that would naturally be measured in terms of area would be 2D. A 1D object would have a length. A 3D object would have volume.

For example, a circle has both a circumference (a length) and an area. Is it both 1D and 2D? Kind of. We’re actually considering two different objects. Let me try to make this precise.

We can consider the set of points in the $xy$-plane that satisfy the equation $x^2 + y^2 = 1$. This is a curve, which we typically call a circle.

Now consider the set of points in the $xy$-plane that satisfies the inequality $x^2 + y^2 \leq 1$. This is a region in the plane, which is also often called a circle, but I’ll call it the inside of a circle, or more often, a disk.

So I’ll say that a circle (the outside curve) is 1D, and a disk is 2D.
1. The term *sphere* has a similar ambiguity. It has a surface area (a 2D attribute), and also a volume (a 3D attribute).

   a. When I say *sphere*, I'll typically mean the outer surface. What is the dimension of a sphere?

   b. I'll usually call the inside of the sphere *a ball*. What is the dimension of a ball?

   c. The earth is roughly a ball. The outer surface is a sphere, and there is a curve on the surface that we call the *equator*. What is the dimension of the equator?

A die is a cube, so it has six faces, and we put the numbers 1, 2, 3, 4, 5, and 6 on the faces so that opposite faces add up to seven. In the picture, the die on the right has a 1, a 3, and a 5 showing. The sum-to-seven rule says that there must be a 6 on the left face, a 4 on the bottom, and a 2 on the back. If you can see three faces of a die, therefore, you know where all the numbers are. The thing we do with dice is roll them, so if we have two dice, and we can rotate them in three dimensions so that the numbers look the same on both dice, then we'll say that the numbers are put on the same way. That way of putting the numbers on will be one orientation. So how many different orientations are there?

1. **Homework/Quiz 01C**

   1. Of the eight configurations of the $xy$-axes, how do they group together in terms of being left- or right-handed?

   2a. Draw six different pictures of one orientation of a die, and the six more of the other.

   2b. Or if you want, draw all the possible pictures with a one on the front, and separate them into the two different orientations.