MA 2233 Lecture 02 - Orientation and Intro to graphing and vectors

Wednesday, August 26, 2015.

Objectives: Start looking at graphing and introduce vectors.

**Orientation**

Let’s consider a simple 3D concept using dice.

A die is a cube (roughly), and it has six faces. We put the numbers 1, 2, 3, 4, 5, and 6 on the faces so that opposite faces add up to seven. Note that a die is a 3D object, but the faces are 2D, the edges are 1D, and the corners are 0D. In the picture, the die on the right has a 1, a 3, and a 5 showing. The sum-to-seven rule says that there must be a 6 on the left face, a 4 on the bottom, and a 2 on the back. If you can see three faces of a die, therefore, you know where all the numbers are. The thing we do with dice is roll them, so if we have two dice, and we can rotate them in three dimensions so that the numbers look the same on both dice, then we’ll say that the numbers are put on the same way. That way of putting the numbers on will be one orientation. So how many different orientations are there?

OK. The answer is that there are two basic orientations for dice. One way of thinking about this is to start with a blank cube. You can put the 1 on any of the faces, and it doesn’t really matter which one, since all the faces are the same. The 2 can’t go on the face opposite the 1, since $1 + 2 \neq 7$. The 2 must go on a face adjacent to the 1. Again, all the adjacent faces are basically the same, so it doesn’t really matter which adjacent face we choose. Essentially, there’s only one way up to this point. Now it’s time to put the 3 on. Since $1 + 3 \neq 7$ and $2 + 3 \neq 7$, we can’t put the 3 opposite the 1 or the 2. That leaves only two possible faces, and either one of these is OK, so there are two ways we can put the 1, 2, and 3 on the die. There aren’t any more choices, because now the 4 must be opposite the 3, the 5 opposite the 2, and the 6 opposite the 1. We’re done. Only two ways.

**Quiz 02A**

1. Imagine that you’re a 2D person living in the $xy$-plane. A 2D die might be a square (I’ll call it a square disk). We can number the faces (which are now 1D) 1, 2, 3, and 4. We’ll follow a sum to 5 rule.
   
   a. Would it be possible to have the numerals 1, 2, 3, and 4 written on a die like this?
   
   b. Let’s assume that little colored line segments could act like dots. How many orientations would 2D die have? Remember that the die have to stay in the $xy$-plane, but they can rotate.
   
   c. Now suppose the $xy$-plane is your desk, and the die is a piece of paper that you can pick up. How many orientations now?

2. See if you can handle this. A 3D die has two orientations. But that’s only if it has to stay in $xyz$-space. If $xyz$-space is sitting in a 4D space, you could pull a die out of $xyz$-space, turn it over, and put it back reversing its orientation. Can you visualize that?
Graphing in 3D

Instead of the 2D $xy$-plane, we have the 3D $xyz$-space. One more dimension means one more variable, $z$. The question becomes, where do we put the $z$-axis? It should be perpendicular to the other two. Like dice, there are basically two ways we can do it. The standard way is shown below.

Notice that the $xy$-plane is like the floor of room extended to infinity, and the $z$-axis goes up. Look at the lower-left-front corner of the room. The $x$-axis comes toward you, the $y$-axis goes to the right, and $z$ goes up. If you turn to the right and look down, you’ll see the standard orientation for the $xy$-plane.

Quiz 02B

In the picture above, the positive $x$-, $y$-, and $z$-axes are shown. We could have chosen for them to go in the opposite direction (or make the ones shown negative). That makes 12 possible ways to label the axes. Find them all, and determine which have the same orientation as the standard one, up to rotations.

Coordinates

Plotting points in $xyz$-space is similar, in theory, to plotting points in the $xy$-plane. For example, coordinates in 2D are written in the form $(x, y)$, and to find a specific point like $(3, 2)$, we start at the origin (where the axes cross), we move 3 units in the positive $x$-direction, and then we move 2 units in the positive $y$-direction.

To plot a point $(3, 2, 1)$, we do the same thing. We move 3 units in the (positive) $x$-direction, 2 units in the $y$-direction, and then 1 unit in the $z$-direction.

There is a lot of ambiguity in the picture, and you have to fill in some of the information in your mind. We’ll always have to do that, but there are “artistic” tricks that can help us a little. We’ll keep talking about this, but that’s the basic idea.
1. The distance between \((0,0)\) and \((x,y)\) in the \(xy\)-plane is given by \(\sqrt{x^2 + y^2}\). So all the points that satisfy the equation \(\sqrt{x^2 + y^2} = 2\) are all the points a distance 2 from the origin. Describe this set of points geometrically. Note that you can square both sides of this equation and get \(x^2 + y^2 = 4\).

2. In three dimensions, the formula looks the same. The distance between \((0,0,0)\) and \((x,y,z)\) is \(\sqrt{x^2 + y^2 + z^2}\). How far is the point \((1,2,3)\) from the origin?

3. The points that satisfy the equation \(x^2 + y^2 + z^2 = 4\) are the points a distance 2 from the origin in \(xyz\)-space. What geometric object is this?

4. There are six points on the surface described in Problem 3 that lie on one of the coordinate axes (i.e. the \(x\)-axis, the \(y\)-axis, or the \(z\)-axis). What are they?

5. There are two points of the form \((1,1,z)\) that lie on the surface from Problem 3. What are they? Note that these lie directly above and below the point \((1,1)\) on the \(xy\)-plane.

Answers on next page.
1) Circle of radius 2 centered at the origin.

3) A sphere of radius 2 centered at the origin.

5) $1^2 + 1^2 + z^2 = 4$, so $z^2 = 2$, and $z = \pm \sqrt{2}$. The points are $(1, 1, \pm \sqrt{2})$. 