Objectives: Review how exponentiation distributes over multiplication.

Exponents distribute over multiplication

Consider the expression \((ab^2)^3\). We can rearrange the factors as follows.

\[
(1) \quad (ab)^3 = ab \cdot ab \cdot ab = a \cdot a \cdot a \cdot b \cdot b \cdot b = a^3 \cdot b^3.
\]

If we have a product raised to an exponent, the exponent distributes to every factor. In other words, exponentiation distributes over multiplication (but not addition). In our hierarchy, exponents distribute over multiplication, and multiplication distributes over addition. Each operation distributes over the operation one level down.

Here are some examples.

\[
(2) \quad (2x^2)^4 = 2^4 \cdot (x^2)^4 = 16x^8
\]

\[
(3) \quad (3 \cdot x^3)^3 = 3^3 \cdot x^9 = 27x^9
\]

\[
(4) \quad (3xy^2)^2 = 9x^2y^4
\]

Exponents also distribute over division

Remember that division goes with multiplication, so we should expect that exponents will distribute over division. For example, it is true that

\[
(5) \quad \left(\frac{2}{x}\right)^3 = \frac{2^3}{x^3} = \frac{8}{x^3}
\]

In general, if we have an expression that involves only multiplications and divisions, then an exponent on that expression will distribute over every factor in the numerator and denominator.

Here are some examples.

\[
(6) \quad \left(\frac{x^2}{3}\right)^2 = \frac{x^4}{3^2} = \frac{x^4}{9}
\]

\[
(7) \quad \left(\frac{2}{3x^2}\right)^3 = \frac{2^3}{3^3x^6} = \frac{8}{27x^6}
\]

\[
(8) \quad \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}
\]
Quiz 12, Part I

Expand out the following. You might want your calculator to simplify the constants.

1. \((2x^3)^4 = 2^4 \cdot (x^3)^4 = 16x^{12}\)
2. \((3x^4)^2 = 3^2 \cdot (x^4)^2 = 9x^8\)
3. \((4x^5)^3 = 4^3 \cdot (x^5)^3 = 64x^{15}\)
4. \((4x^3)^4 = 4^4 \cdot (x^3)^4 = 256x^{12}\)
5. \((3x^2)^6 = 3^6 \cdot (x^2)^6 = 729x^{12}\)

6. \(\left( \frac{3}{x} \right)^3 = \frac{3^3}{x^3} = \frac{27}{x^3}\)
7. \(\left( \frac{x}{2} \right)^4 = \frac{x^4}{2^4} = \frac{x^4}{16}\)
8. \(\left( \frac{2x}{3} \right)^2 = \frac{2^2x^2}{3^2} = \frac{4x^2}{9}\)
9. \(\left( \frac{4}{3x} \right)^3 = \frac{4^3}{3^3x^3} = \frac{64}{27x^3}\)
10. \(\left( \frac{x^3}{2} \right)^2 = \frac{(x^3)^2}{2^2} = \frac{x^6}{4}\)

Exponents do not distribute over addition

You might be very tempted to distribute an exponent over an addition, but you can’t.

\((x + 2)^2 \neq x^2 + 2^2\)

We already know that

\((x + 2)^2 = (x + 2)(x + 2) = x^2 + 2x + 2x + 4 = x^2 + 4x + 4.\)

Scientific notation

It is common, and our brains seem to be comfortable with this, to work with numbers like

\((11)\)

\(27.236\)

In particular, it gets harder when we try to keep track of more than four or five digits. Really big and really small numbers are awkward. For example, look at

\((12)\)

\(0.0000000007243\)

You probably have to look pretty closely and count the number of zeros, although the 7243 at the end is not too bad.

In order to make really big and really small numbers like this a bit easier to read, we sometimes use something called scientific notation. For example, we would write the number above like

\((13)\)

\(7.243 \times 10^{-10}\)

Multiplying by 10 is relatively easy, and the negative exponent means were dividing by 10 factors of 10. After you get used to it, instead of counting zeros, you can just read the exponent \(-10\).
Here's another example.

(14) \[5.662 \times 10^{12} = 5.662 \times 1,000,000,000,000 = 5,662,000,000,000\]

The exponent on the 10 corresponds to moving the decimal point that number of places. You can keep things straight by remembering that positive exponents on the 10 go with big numbers, and negative exponents go with really small numbers. In scientific notation, there is always exactly one non-zero digit to the left of the decimal point. Converting back and forth basically involves moving the decimal point and keeping track of how many places.

To convert 172,246,000,000 to scientific notation, we need to move the decimal point (which is after that last zero) to behind the 1. That means moving the decimal point 11 places. Therefore,

(15) \[172,246,000,000 = 1.72246 \times 10^{11}\]

**Quiz 12, Part II**

Convert to or from scientific notation.

11. \[527,100,000 = 5.271 \times 10^8\]
12. \[0.00000791 = 7.91 \times 10^{-7}\]
13. \[3.33 \times 10^{10} = 33,300,000,000\]
14. \[7.81 \times 10^{-4} = 0.000781\]
Homework 12

Expand out the following. You might want your calculator to simplify the constants.

1. \((2x^5)^2\)
2. \((4x^3)^6\)
3. \((5x^7)^2\)
4. \((4x)^9\)
5. \((\frac{2}{x})^5\)
6. \((\frac{x^2}{3})^4\)
7. \((\frac{3x}{2})^2\)
8. \((\frac{5}{3x^4})^3\)

For problems 9-15, True or False, are the two things actually equal?

9. \((2x)^3 = 2^3 \cdot x^3?\)
10. \((x + 4)^3 = x^3 + 4^3?\)
11. \((x^2 + 1)^2 = x^4 + 1^2?\)
12. \((x - 3)^3 = x^3 - 3^3?\)
13. \((x + 1)^4 = x^4 + 1?\)

Convert to or from scientific notation.

14. \(328,400,000,000\)
15. \(0.000000012225\)
16. \(7.335 \times 10^{12}\)
17. \(8.446 \times 10^{-7}\)
18. \(7.089 \times 10^{-11}\)
19. \(1.0003 \times 10^8\)
20. \(0.0000000012003\)