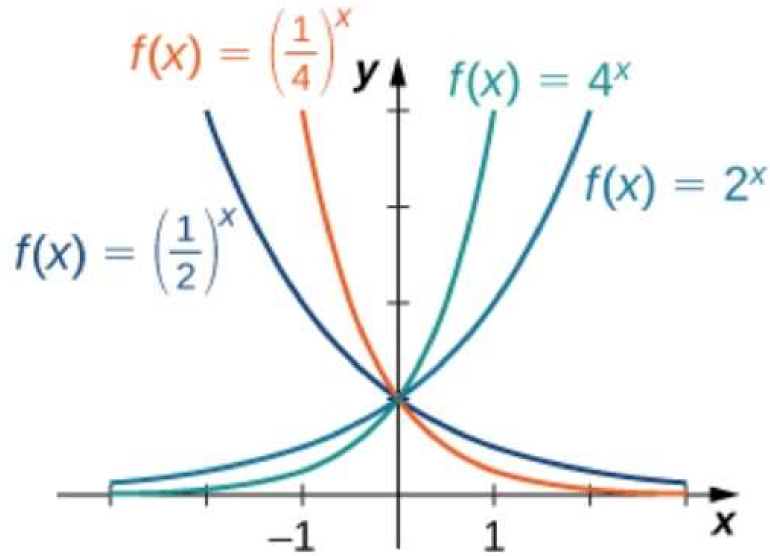

Figure 1.44 If $b > 1,$ then b^x is increasing on $(-\infty, \infty).$ If $0 < b < 1,$ then b^x is decreasing on $(-\infty, \infty).$

Rule: Laws of Exponents

For any constants $a > 0, b > 0,$ and for all x and y,

1. $b^x \cdot b^y = b^{x+y}$
2. $\frac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$
4. $(ab)^x = a^x b^x$
5. $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

$(1 + 1/m)^m$ as $m \to \infty$, using a table of values (Table 1.12).

m	10	100	1000	10,000	100,000	1,000,000
$\left(1 + \frac{1}{m}\right)^m$	2.5937	2.7048	2.71692	2.71815	2.718268	2.718280

Table 1.12 Values of $\left(1 + \frac{1}{m}\right)^m$ as $m \to \infty$

Looking at this table, it appears that $(1 + 1/m)^m$ is approaching a number between 2.7 and 2.8 as $m \to \infty$. In fact, $(1 + 1/m)^m$ does approach some number as $m \to \infty$. We call this **number *e***. To six decimal places of accuracy,

$$e \approx 2.718282.$$

The letter e was first used to represent this number by the Swiss mathematician Leonhard Euler during the 1720s.

Logarithmic Functions

Using our understanding of exponential functions, we can discuss their inverses, which are the logarithmic functions. These come in handy when we need to consider any phenomenon that varies over a wide range of values, such as pH in chemistry or decibels in sound levels.

The exponential function $f(x) = b^x$ is one-to-one, with domain $(-\infty, \infty)$ and range $(0, \infty)$. Therefore, it has an inverse function, called the *logarithmic function with base* b. For any $b > 0, b \neq 1,$ the logarithmic function with base b, denoted $\log_b,$ has domain $(0, \infty)$ and range $(-\infty, \infty),$ and satisfies

$$\log_b(x) = y \text{ if and only if } b^y = x.$$

$$\begin{aligned} &\log_2(8) = 3 && \text{since} && 2^3 = 8, \\ &\log_{10}\left(\frac{1}{100}\right) = -2 && \text{since} && 10^{-2} = \frac{1}{10^2} = \frac{1}{100}, \\ &\log_b(1) = 0 && \text{since} && b^0 = 1 \text{ for any base } b > 0. \end{aligned}$$

Furthermore, since $y = \log_b(x)$ and $y = b^x$ are inverse functions,

$$\log_b(b^x) = x \text{ and } b^{\log_b(x)} = x.$$

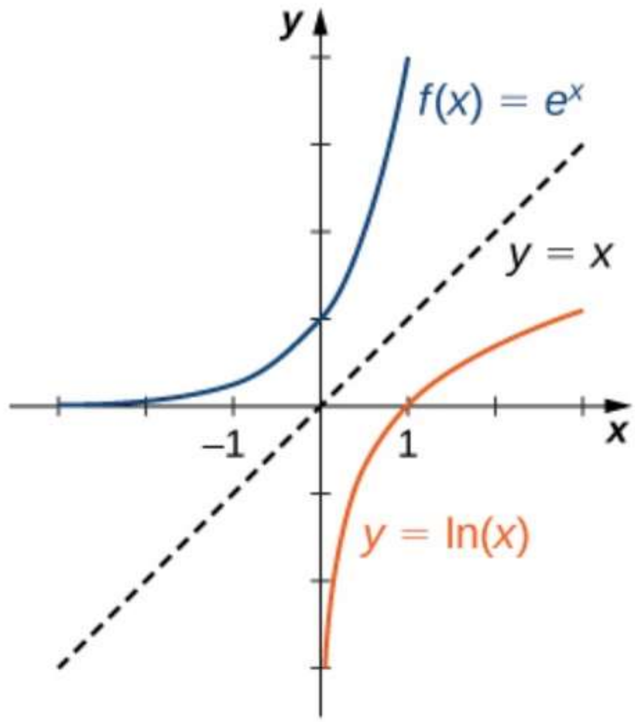

Figure 1.46 The functions $y = e^x$ and $y = \ln(x)$ are inverses of each other, so their graphs are symmetric about the line $y = x$.

Rule: Properties of Logarithms

If $a, b, c > 0, b \neq 1,$ and r is any real number, then

1. $\log_b(ac) = \log_b(a) + \log_b(c)$ (Product property)
2. $\log_b\left(\frac{a}{c}\right) = \log_b(a) - \log_b(c)$ (Quotient property)
3. $\log_b(a^r) = r\log_b(a)$ (Power property)

Rule: Change-of-Base Formulas

Let $a > 0, b > 0,$ and $a \neq 1, b \neq 1.$

1. $a^x = b^{x\log_b a}$ for any real number x.

   If $b = e,$ this equation reduces to $a^x = e^{x\log_e a} = e^{x\ln a}$.

2. $\log_a x = \frac{\log_b x}{\log_b a}$ for any real number $x > 0$.

   If $b = e,$ this equation reduces to $\log_a x = \frac{\ln x}{\ln a}$.