

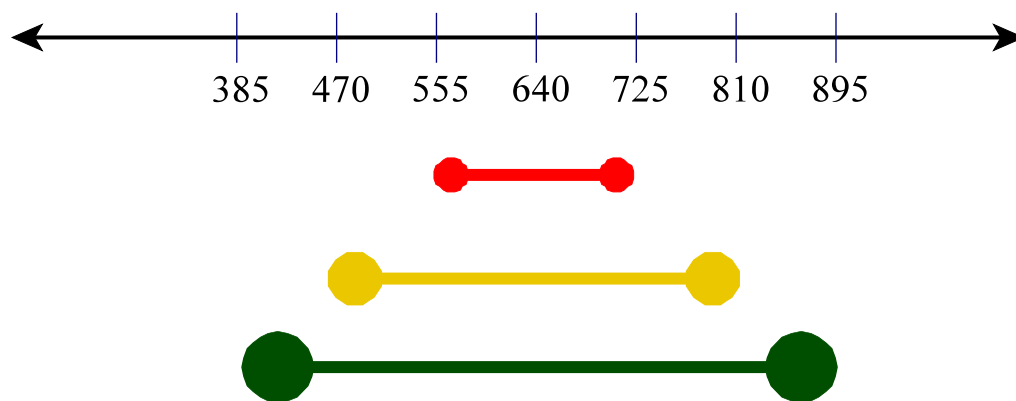
Question.

The average college student produces 640 pounds of solid waste a year, including 500 cups and 320 pounds of paper. If the standard deviation is approximately 85 pounds within what weight limits will at least 88.89% of all garbage lie?

Let's use a number line. The mean is 640, and we have an SD of 85. So. . . I'll put in 640 at the middle and 'skip' by 85's so to speak. Why 640? It is our 'best' measurement of the middle. Skipping by 85's happens because the SD is, in some sense, the 'best' measurement of spread. (Think z-scores.)

We do this for quite a few things, so it's important that you "see" the picture.

Here's the picture.



640 is the middle.

555 up to 725, is all the weights within one SD of the mean. (Red highlight.)

470 up to 810, is all the weights within two SD's of the mean. (Yellow highlight.)

385 up to 895, is all the weights within three SD's of the mean. (Green highlight.)

**Empirical rule.** If the parent distribution has a standard deviation and is reasonably close to being bell-shaped then about 68% is in the red, 95% is in the yellow, and 99.7% is in the green.

PS: It's pretty tough to truly verify that the distribution is close enough to being bell-shaped without consulting a statistician.

**Chebyshev's rule.** If the parent distribution has a standard deviation, then no matter what the parent distribution's shape is, at least  $1 - 1/k^2$  percent of the distribution lies within  $k$  SD's of the mean, for  $k > 1$ . To build a few of them:

When  $k = 1.5$  we have  $1 - 1/k^2 = 0.555555 = 55.5\%$  (rounding.)

When  $k = 2$  we have  $1 - 1/k^2 = 0.75 = 75\%$

When  $k = 3$  we have  $1 - 1/k^2 = 0.888888 = 88.9\%$  (rounding.)

Using Chebyshev's rule we have 75% is in the yellow, and 88.9% is in the green.