A worked out and fairly complicated example.

Box 1 contains 4 marbles: 3 Blue, and 1 Red. Box 2 also contains 4 marbles: 2 Red and 2 Blue.


The experiment is multistage. There are a total of 3 draws. First, draw twice from box 1 without replacement. Now place the two marbles drawn from box 1 and put them into box 2. The third and final draw is made from box 2 .

Notation: I'll write "BBR" to mean Blue on $1^{\text {st }}$ draw, Blue on $2^{\text {nd }}$ draw and Red on $3^{\text {rd }}$ draw.
Find the following probabilities:
a. $\mathrm{P}(\mathrm{BBR})$
b. $\mathrm{P}(\mathrm{RRB})$
c. P(exactly 2 reds out of the 3 draws)
d. $\mathrm{P}(\mathrm{R}$ from box $2 \mid$ two blue were drawn from box 1$)$
e. P (exactly one R from box $1 \mid \mathrm{R}$ was drawn from box 2 )

Ponder what these mean for a moment. Try to figure them out on your own if you'd like. When you're ready, we'll start on the next page.

The easiest way to do this problem is to draw a tree diagram to get the sample space and probabilities of each outcome for the experiment.

## 1. Draw the tree diagram for this experiment.

Step 1. Here's a tree for the first draw from box 1, complete with the probabilities for each outcome. Notice that I don't draw three lines for the blue marble.


B R

Step 2. The second draw from box 1. Notice that all the new probabilities are conditional, based on the first draw.



B R


Step 3: Now we add the draw from box 2 to the tree.


Step 4: We can now see all the outcomes for this experiment and their probabilities. The outcomes are found by taking all the possible routes from the top to the bottom, and the probabilities are found by multiplying each branch's probability.

From left to right we have:

| Outcome | Probability of Outcome |
| :---: | :---: |
| BBB | $1 / 3=8 / 24$ |
| BBR | $1 / 6=4 / 24$ |
| BRB | $1 / 8=3 / 24$ |
| BRR | $1 / 8=3 / 24$ |
| RBB | $1 / 8=3 / 24$ |
| RBR | $1 / 8=3 / 24$ |

(I have the probabilities reduced and then also all with a common denominator. This will ease some of the computations I'm going to do next, but it isn't technically necessary.)

Now I attempt the exercises.
a. $\mathrm{P}(\mathrm{BBR})$. This is a basic outcome, so we can read it right off the chart. $1 / 6$.
b. $\mathrm{P}(\mathrm{RRB})$. This can't happen. Its probability is zero.
c. P(exactly 2 reds out of the 3 draws). The probability of an event is the sum of all the basic outcomes contained in that event. I check each outcome to see if it satisfies the event.

| BBB | $1 / 3=8 / 24$ |
| :--- | :--- |
| BBR | $1 / 6=4 / 24$ |
| BRB | $1 / 8=3 / 24$ |
| BRR | $\mathbf{1 / 8}=\mathbf{3} / \mathbf{2 4}$ |
| RBB | $1 / 8=3 / 24$ |
| RBR | $\mathbf{1 / 8}=\mathbf{3} / \mathbf{2 4}$ |

The two highlighted outcomes are the only two in the event. Adding their probabilities gives us a total of $6 / 24=1 / 4$.
d. $\mathrm{P}(\mathrm{R}$ from box $2 \mid$ two blue were drawn from box 1$)$

If you know how to read this question correctly, it isn't too bad. In English it is asking "what is the probability of drawing a red from box 2 given that two blue marbles were drawn from box 1 ." We computed this to construct our tree in step 3 . It is $2 / 6$ or $1 / 3$.
e. P (one R from box $1 \mid R$ was drawn from box 2 ).

This is an example of conditional probability where you almost need to rely on the formula. (Read it in English, it feels backwards.)

For any two events A and B , we have $\mathrm{P}(\mathrm{A} \mid \mathrm{B})=\mathrm{P}(\mathrm{A}$ and B$) / \mathrm{P}(\mathrm{B})$.
$\mathrm{P}(\mathrm{A}$ and B$)=\mathrm{P}($ one R from box 1 and R was drawn from box 2$)$. I will highlight all the outcomes that satisfy this:

| BBB | $1 / 3=8 / 24$ |
| :--- | :--- |
| BBR | $1 / 6=4 / 24$ |
| BRB | $1 / 8=3 / 24$ |
| BRR | $\mathbf{1 / 8}=\mathbf{3 / 2 4}$ |
| RBB | $1 / 8=3 / 24$ |
| RBR | $\mathbf{1 / 8}=\mathbf{3 / 2 4}$ |

So the probability of $\mathrm{P}(\mathrm{A}$ and B$)=3 / 24+3 / 24=6 / 24$.
Now I compute $P(B)$ in the same way:

| BBB | $1 / 3=8 / 24$ |
| :--- | :--- |
| BBR | $\mathbf{1 / 6}=\mathbf{4} / \mathbf{2 4}$ |
| BRB | $1 / 8=3 / 24$ |
| BRR | $\mathbf{1 / 8}=\mathbf{3 / 2 4}$ |
| RBB | $1 / 8=3 / 24$ |
| RBR | $\mathbf{1 / 8}=\mathbf{3 / 2 4}$ |

Thus, $\mathrm{P}(\mathrm{B})=4 / 24+3 / 24+3 / 24=10 / 24$.
The answer is then $\mathrm{P}(\mathrm{A} \mid \mathrm{B})=\mathrm{P}(\mathrm{A}$ and B$) / \mathrm{P}(\mathrm{B})=6 / 24 \div 10 / 24=6 / 24 * 24 / 10=6 / 10=3 / 5$.

