

More conditional probability examples.

There are cards with two sides. Suppose that you draw a card at random and with equal probability. You are only allowed to see the front of the card.

<u>Card</u>	<u>Front</u>	<u>Back</u>
1.	A	X
2.	A	X
3.	B	Y
4.	B	Y

I will abbreviate events in this pretty obvious way: A = “Your card has an A on it.”

Find:

- 1.1. $P(X)$
- 1.2. $P(X | A)$
- 1.3. $P(X | B)$
- 1.4. $P(A \text{ and } X)$
- 1.5. $P(B \text{ and } X)$
- 1.6. $P(A \text{ or } X)$
- 1.7. $P(B \text{ or } X)$

Answers:

- 1.1. There are 4 cards, 2 have an X. So, $P(X) = 2/4 = 50\%$.
- 1.2. There are 2 cards with an A on them, and they both have an X. $P(X | A) = 2/2 = 100\%$.
- 1.3. There are 2 cards with a B on them, and none of them have an X. $P(X | B) = 0/2 = 0\%$
- 1.4. Of the 4 cards, exactly 2 of them have both an A and an X. $P(A \text{ and } X) = 2/4 = 50\%$
- 1.5. Of the 4 cards, none of them have both a B and an X. $P(B \text{ and } X) = 0/4 = 0\%$
- 1.6. Of the 4 cards, exactly 2 of them have an A, an X, or both. $P(A \text{ or } X) = 2/4 = 50\%$
- 1.7. Of the 4 cards, 4 of them have a B, an X, or both. $P(B \text{ or } X) = 4/4 = 100\%$

Okay, let's turn it up a little bit by using a different deck.

<u>Card</u>	<u>Front</u>	<u>Back</u>
1.	A	X
2.	A	X
3.	A	Y
4.	B	X
5.	B	Y
6.	B	Y

Find:

- 2.1. $P(X)$
- 2.2. $P(X | A)$
- 2.3. $P(X | B)$
- 2.4. $P(A \text{ and } X)$
- 2.5. $P(B \text{ and } X)$
- 2.6. $P(A \text{ or } X)$
- 2.7. $P(B \text{ or } X)$

Answers (rounded to 3 digits):

- 2.1. There are 6 cards, 3 have an X. So, $P(X) = 3/6 = 50\%$.
- 2.2. There are 3 cards with an A on them, and 2 have an X. $P(X | A) = 2/3 = 66.667\%$.
- 2.3. There are 3 cards with a B on them, and one of them has an X. $P(X | B) = 1/3 = 33.333\%$
- 2.4. Of the 6 cards, exactly 2 of them have both an A and an X. $P(A \text{ and } X) = 2/6 = 33.333\%$
- 2.5. Of the 6 cards, one of them has both a B and an X. $P(B \text{ and } X) = 1/6 = 16.667\%$
- 2.6. Of the 6 cards, 4 of them have an A, an X, or both. $P(A \text{ or } X) = 4/6 = 66.667\%$.
- 2.7. Of the 6 cards, 5 of them have a B, an X, or both. $P(B \text{ or } X) = 5/6 = 83.333\%$