

The Cooties Research Institute.

You've Got Cooties! A certain medical test has the following reliability: if a person has cooties the test will show positive with probability 0.95. If a person does not have cooties, the test will show negative with probability 0.6. Overall, 10% of the population has cooties. Give your answers as percentages with two decimal digits, e.g. 93.42%.

C = person has cooties. F = person is cooties "free". P = test is positive for the disease. N = test is negative for the disease.

Statement: If a person has cooties the test will show positive with probability 0.95.

English is *so* twisted. Let me transform the statement.

"If a person has cooties the test will show positive with probability 0.95"
= "Prob(If a person has cooties the test will show positive) = 0.95"
= "Prob(the test will show positive given a person has cooties) = 0.95"
= "Prob(P | C) = 0.95."

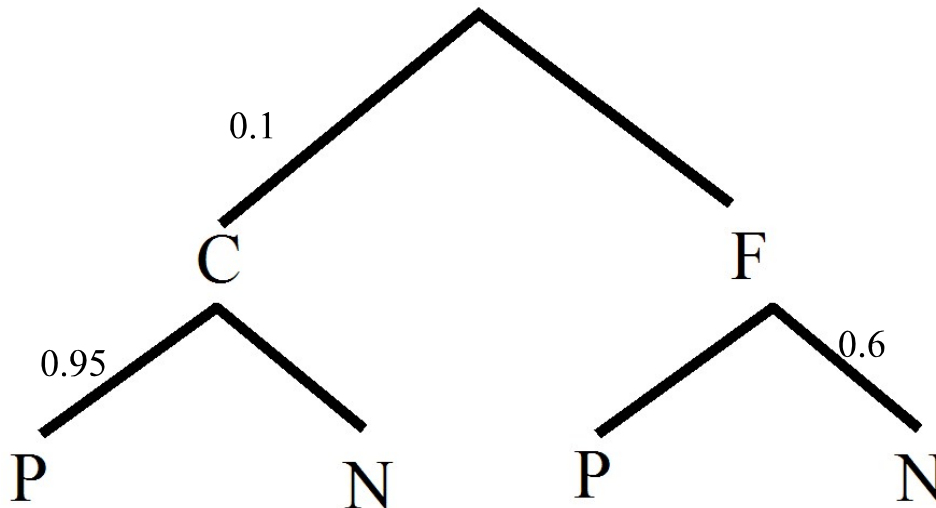
Statement: If a person does not have cooties, the test will show negative with probability 0.6.

"If a person does not have cooties, the test will show negative with probability 0.6."
="Prob(If a person does not have cooties, the test will show negative) = 0.6."
="Prob(the test will show negative given the person does not have cooties) = 0.6"
="Prob(N | F) = 0.6"

Statement: Overall, 10% of the population has cooties.

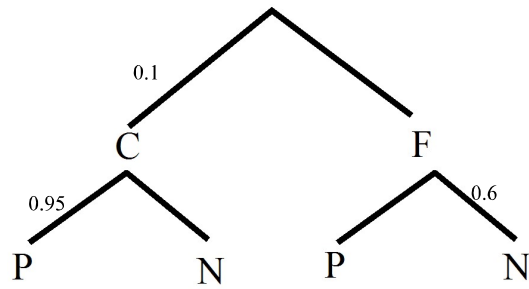
$$\text{Prob}(C) = 0.10.$$

Here's the score so far:



So what exactly is that thing over there?

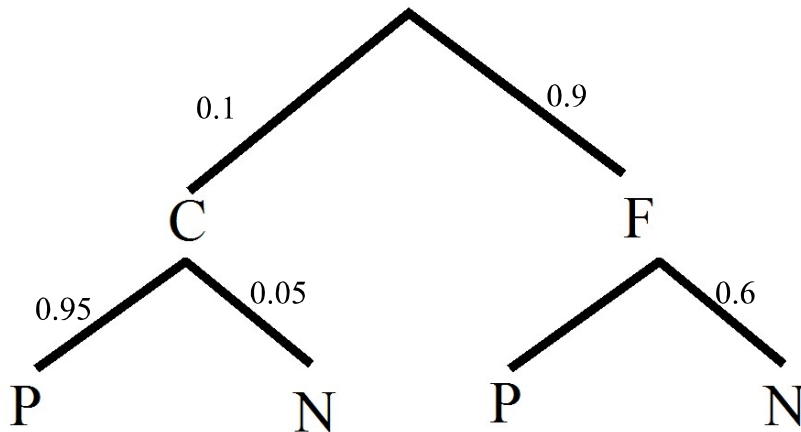
Read it from top to bottom. Each time you go down a level, you've added a characteristic of sorts.



There are two types of people in this world, people with cooties, C, and people without cooties, F.

$P(C) = 0.1$ Therefore $P(F) = 0.9$.

If a person has cooties, we're on the lower left in that picture. **In a tree, all the new probabilities are conditional, based on what's above them.** That's why the 0.95 is there. It represents the probability of testing positive given that the person truly has cooties. So what's the probability of testing negative given that the person truly has cooties? It has to be 0.05. (Can you see it?)



Try and get the last one on your own.

So the last entry in the tree must be 0.4. (If you have a person that does not have cooties they must test either positive or negative. If the probability of the latter is 0.6 then the probability of the former must be 0.4.)

From the tree we see there are 4 types of people in this world.

C and P
C and N
F and P
F and N

Moreover, we have:

$\text{Prob}(C \text{ and } P) = 0.095$
 $\text{Prob}(C \text{ and } N) = 0.005$
 $\text{Prob}(F \text{ and } P) = 0.36$
 $\text{Prob}(F \text{ and } N) = 0.54$

You can throw away the tree now. We have the sample space and the probabilities for all basic outcomes. We can answer any question from what's on this page alone.

So what are the odds that a person that tests positive actually has cooties?

When you've discombobulated the English there I hope you see this as the goal: $\text{Prob}(C | P)$. Now we go back to the definition of conditional probability. . .

$\text{Prob}(C | P) = \text{Prob}(C \text{ and } P) / \text{Prob}(P)$.

To find $\text{Prob}(P)$, add all basic outcomes that satisfy the event. That is $0.095 + 0.36$. I think you can take it from here. . .