MATH 125 Probability Homework

Problem 1. Assume that $P(A) = 0.4$ and $P(B) = 0.3$. for all parts of this problem. Find the following probabilities:

a.) What is $P(\overline{B})$?

$P(\overline{B}) = 1 - P(B) = 1 - 0.3 = 0.7$

b.) Given that P(A and B) = 0.1, find P(A or B).

$$
\begin{aligned}
P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\
&= 0.4 + 0.3 - 0.1 \\
&= 0.6
\end{aligned}
$$

c.) If A and B are mutually exclusive, what is P(A and B)?

By def of mutually exclusive, that is zero.

d.) If A and B are mutually exclusive, what is P(A or B)?

$$
\begin{aligned}
P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\
&= 0.4 + 0.3 - 0.0 \\
&= 0.7
\end{aligned}
$$

e.) Given that P(A or B) = 0.6, find P(B|A).

Two steps here.

$$
\begin{aligned}
P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\
0.6 &= 0.4 + 0.3 - P(A \text{ and } B) \\
0.1 &= P(A \text{ and } B)
\end{aligned}
$$

Now we use:

$$
\begin{aligned}
P(A \text{ and } B) &= P(A)\ P(B|A) \\
0.1 &= 0.4\ P(B|A) \\
0.25 &= P(B|A)
\end{aligned}
$$

f.) Given that P(A|B) = 0.4 what is P(A and B)?

$$
\begin{aligned}
P(A \text{ and } B) &= P(B)\ P(A|B) \\
&= 0.3 * 0.4 \\
&= 0.12
\end{aligned}
$$

g.) If A and B are independent, what is P(A and B)?

$$
\begin{aligned}
P(A \text{ and } B) &= P(B)\ P(A) \quad \text{since they are ind.} \\
&= 0.3 * 0.4 \\
&= 0.12
\end{aligned}
$$

h.) If A and B are independent, what is P(A or B)?

As in part g, P(A and B)= 0.12. Now use the addition rule:

$$
\begin{aligned}
P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\
&= 0.4 + 0.3 - 0.12 \\
&= 0.58
\end{aligned}
$$

Problem 2. Assume that $P(A) = 0.6$ and $P(B) = 0.5$ for all parts of this problem.

a.) If A and B are independent, what is the probability of P(A and B)?

P(A and B)=P(B) P(A) = 0.6*0.5 = 0.3

b.) Can A and B be mutually exclusive? Justify your answer.

If they are, then P(A or B) = P(A) + P(B). (Do you know why?) This presents a problem, for then P(A or B) = 1.1. So the answer is no.

Problem 3. A box contains three cokes and two beers (root beer of course). Julie draws at random twice **without** replacement from the box. (Any draw is equally likely)

a.) What is the probability that she gets at least one beer?

From a tree diagram, (2/5)(1/4) + (2/5)(3/4) + (3/5)(2/4).

b.) What is the probability that she drew a coke on the first draw given that she drew a beer on the second draw?

Let A be "she drew a coke on the first draw" and B be "she drew a beer on the second draw".
We want to find P(A|B). Recall P(B)P(A|B) = P(A and B). Find P(B) and P(A and B) from the tree diagram. They are 0.4 and 0.3, respectively. Thus P(A|B) = 0.75.

c.) Are the events "Julie draws a beer on the first draw" and "Julie draws a beer on the second draw" independent?

No. To show it, (which would be required on a test) show that P(A and B) isn't the same as P(A)P(B).

Problem 4. A box contains three cokes and two beers. James draws at random twice **with** replacement from the box. (Any draw is equally likely)

a.) What is the probability that he gets at least one beer?

$1-(3/5)^2$

b.) What is the probability that he drew a coke on the first draw given that he drew a beer on the second draw?

3/5

c.) Are the events "James draws a beer on the first draw" and "James draws a beer on the second draw" independent?

Yes. The two trials are independent as they are with replacement.

Problem 5. Draw from the box containing 1,2,3 and 4 twice without replacement.

a.) Let A be the event that the sum of the draws is even. Let B be the event that the first draw is odd. Find the probabilities of each of these events.

Here I would draw a square to see the sample space as I did in class with a pair of dice, but omitting the impossible outcomes (1,1), (2,2) (3,3) and (4,4). A tree works too.

P(A) = 3/12 = 1/4

P(B) = 6/12 = 1/2

b.) Are A and B mutually exclusive?

No. P(A and B) = 2/12 = 1/6.

c.) Are A and B independent?

No. P(A and B) = 1/6 which isn't the same as P(A)P(B) = (1/2)(1/4) = 1/8.

Problem 8. Janice wants to become a police officer. She must pass a physical exam and then a written exam. Records show the probability of passing the physical exam is 0.85 and that once the physical exam is passed the probability of passing the written exam is 0.60.

a) What is the probability that Janice passes both exams? 0.85*0.60 = 0.51.
b) What is the probability that Janice fails the written exam if she has already passed the physical exam? 0.4

Problem 9. Suppose a missile defense grid can shoot down 95% (a complete technological miracle) of all incoming nuclear missiles. (Assume that nuclear warheads are just part of a missile for this problem.) What is the probability that out of 10 incoming nuclear missiles, all of them are shot down? You may assume that shooting a missiles are independent events.

(Getting political here: these are absurd assumptions. Tracking multiple ICBM's simultaneously would undoubtably lower the chance of countering more than one of them. The real-world probability of shooting down an ICBM is certainly much lower than 25% in extremely controlled situations.)

Geez, who let this guy out of the math dept and onto the web, lol. Anyway, it is $(.95)^{10}$ which is about 59.874% of the time.