Lab 1

Figure 1: The lines $y = 1$ and $x = 1$ in the $xy$-plane.

**Question 1.** The lines $y = 1$ and $x = 1$ are shown in Figure 1. What is the angle between these lines? How would you describe the relationship between the line $y = 1$ and the negative $x$-axis? The line $x = 1$ and the negative $y$-axis? Imagine cutting along the negative $x$- and $y$-axes, removing the third quadrant, and then taping the negative $x$- and $y$-axes together. Near the tape, how would you describe the relationship between the lines $y = 1$ and $x = 1$?

**Question 2.** Now actually do the cutting and taping of Question 1. What would you call the resulting surface? You had to bend the paper a bit to get the two edges taped together, but did you have to stretch or tear the paper to do this? For measurements along the paper, did distances between objects drawn on the paper change when you brought the two edges together? The lines $y = 1$ and $x = 1$ are now curves on this new surface. Are they geodesics?

**Question 3.** On your surface from Question 2, cut along the positive $x$-axis. You should be able to flatten the surface after doing this. The line $y = 1$ was cut into two pieces. How would you describe the relationship between the two pieces of the line when the paper is flat?

Figure 2: The line $y = x - 1$ in the $xy$-plane.

**Question 4.** The line $y = x - 1$ is shown in Figure 2. If you remove the third quadrant as you did in Questions 1 and 2, this line just stops at the cut. On the surface, it should continue on into what was the second quadrant. Find the equation of another line in the $xy$-plane such that when you make the cone, the line $y = x - 1$ continues onto this line and together they form a geodesic.
Question 5. The line $y = -x - 1$ isn’t a geodesic after you make the cone, because it will have a sharp angle at the cut. What is the measure of this angle? What line or lines should the two pieces continue onto to make geodesics?
Lab 1 – Answers and Notes

Question 1. The two lines are perpendicular, so they make angles of 90° or $\frac{\pi}{2}$ radians. The line $y = 1$ is parallel to the negative $x$-axis, and $x = 1$ is parallel to the negative $y$-axis. After taping the edges together, the negative $x$- and $y$-axes are the same, so the two lines must be parallel to each other (at least in this part of the surface). This can be seen in Figure 3. It might be better to say that parts of these two lines are equidistant.

Figure 3: A paper model for Question 2.

Question 2. See Figure 3. I would call this surface a cone. You should have been able to do this without any tearing or stretching. As a result, distances along the paper remain the same, although distances in space will change. Yes, both lines become geodesics.

Question 3. I would say that the two pieces of $y = 1$ are now perpendicular.

Question 4. As this line crosses the negative $y$-axis, the part in the third quadrant makes a 45° angle with the negative $y$-axis. The continuation of this line on the cone should make the same angle with the negative $x$-axis. Therefore, this other line must have slope $m = -1$. It also must intersect the $x$-axis at $-1$, so the equation must be $y = -x - 1$.

Figure 4: The continuation lines in Question 5 are dotted.

Question 5. See Figures 4 and 5. When you remove the third quadrant and tape the edges together, the remaining parts of the line $y = -x - 1$ come together at an angle measuring 90° along the surface. For geodesics this angle should be 180° (a 180° angle is called a straight angle for good reason). The part of this line in the second quadrant makes a 45° angle with the negative $x$-axis, so its continuation should make a 135° angle with the negative $y$-axis. In the plane, the continuation must therefore have slope 1 and $y$-intercept $-1$, and so it must have equation $y = x - 1$. Similarly, the part in the third quadrant must continue onto the line $y = x + 1$. 
Figure 5: On the cone, the dotted lines continue the solid lines smoothly across the cut.