1. Suppose we have five integers related by the equation \( c = sa + tb \).
   a. If an integer \( d \) divides both \( a \) and \( b \), must \( d \) also divide \( c \)?
   b. If \( d \) divides both \( c \) and \( a \), must \( c \) divide \( tb \)?
   c. If \( d \) divides both \( c \) and \( a \), \( c \) might not divide \( b \). Find a counter example.

2a. Use the Euclidean algorithm on 168 and 60.
2b. Use the results of Part a to express \( \text{GCD}(168, 60) \) as a linear combination of 168 and 60.

3a. Using “divides” as your ordering relation, draw a lattice for the factors of 42.
3b. Do the same for 27.
3c. Give me four numbers that have the same factor lattice as 27.

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4. Do the following division problems in \( \mathbb{Z}_{12} \) using the multiplication table. You might have more than one possible answer, give all of them. You also might not have any, just say \( \text{DNE} \) (does not exist) for these.
   a. \( 7 \div 5 \).
   b. \( 3 \div 6 \).
   c. \( 8 \div 10 \).
   d. \( 5 \div 2 \).
   e. \( 2 \div 5 \).
5. If we’re trying to solve an equation like $4x = 6$, we can’t just divide by 4, because 4 does not have a multiplicative inverse. We have to, like some of the problems in Problem 1, just look at the table, and see which elements of $\mathbb{Z}_{12}$ satisfy “4 times $x$ equals 6.” Looking at the table, we see there are no solutions. For the following, find all solutions, if any.

a. $5x = 9.$

b. $3x = 0.$

c. $4x = 8.$

d. $11x = 4.$

e. $9x = 3.$

6. Since every element in $\mathbb{Z}_{12}$ has an additive inverse, subtracting an element from both sides of an equation is fine. Solve the following by subtracting something from both sides of the equation, and then proceeding as in Problem 2.

a. $3x + 7 = 11.$

b. $6x + 4 = 0.$

c. $2x + 10 = 8.$

d. $11x + 5 = 7.$

e. $4x + 3 = 5.$

7. Find the Conway polynomial of the trefoil. Show all your work, and assume that the only Conway polynomials you know are $\nabla_{\text{unknot}} = 1$ and $\nabla_{\text{unlink}} = 0$.

8. I told you that any unlinked link has Conway Polynomial equal to 0. You’re going to demonstrate this. Suppose we have an unlinked link where the two parts of the link are the knots $A$ and $B$. You want to show that the Conway Polynomial must be 0. Use the connect sum of $A$ and $B$ shown below.
\begin{align*}
a \lor (b \lor c) &= (a \lor b) \lor c & a \land (b \land c) &= (a \land b) \land c \\
a \lor b &= b \lor a & a \land b &= b \land a \\
a \lor 0 &= a & a \land 1 &= a \\
a \lor (b \land c) &= (a \lor b) \land (a \lor c) & a \land (b \lor c) &= (a \land b) \lor (a \land c) \\
a \lor \sim a &= 1 & a \land \sim a &= 0
\end{align*}

(1)

\begin{align*}
a \lor 0 &= 0 \\
a \land 1 &= 1 \\
a \land (a \land b) &= a \\
a \lor (a \land b) &= a \\
a \land a &= a \\
a \lor a &= a \\
\text{If } a \lor b &= 1 \text{ and } a \land b = 0, \text{ then } b &= \sim a. \\
\sim (a \land b) &= \sim a \lor \sim b \\
\sim (a \lor b) &= \sim a \land \sim b \\
\sim (\sim a) &= a
\end{align*}

(2)

9a. Use the properties above to simplify \( a \lor (\sim a \land b) \).

9b. Use the properties to simplify \((a \lor b) \land (a \lor \sim b) \land (\sim a \lor \sim b)\).