1. In the figure below is a trefoil $T$ and a figure-eight knot $F$. Draw the connect sum of these two knots $T\#F$.

2. We know that the Conway polynomial of $T$ is $\nabla_T = 1 + x^2$ and $\nabla_F = 1 - x^2$. What is the Conway polynomial for $T\#F$? Show your work and explain what you’re doing.

3. We do not know that the unknot is the only knot with a Conway polynomial $\nabla_K = 1$. There may be others. Suppose $A\#B = U$, where $U$ is the unknot. Do you know what $\nabla_A$ is? Explain how you know.
4. Shown below, the Mystery Knots $A$ and $B$ taken together form an unlinked link $C$. Show using the Conway polynomial equation that $\nabla_C = 0$.

$$\nabla_R - \nabla_L = x\nabla_S$$

5. Draw a picture of a trefoil. Draw two non-trivial loops $a$ and $b$ (your choice). Now draw $a^2b^{-1}$. Loops are oriented, so don’t forget your arrows!
6a. Consider a six-hour clock (we're talking $\mathbb{Z}_6$, and 6 is at the top). Let $j$ represent jumps of $j$ hours. For each $j$, let $k$ be the number of jumps it takes to start at 6 o'clock and get back to 6 o'clock. For each $j = 1, 2, 3, 4, 5, 6$, find the corresponding $k$.

6b. Explicitly describe a process that will find $k$ for any positive integer. You may use language such as “Divide by eight, take the remainder, subtract two, …” You may also refer to your answer in problem 6a.

7. List out all the subgroups of $\mathbb{Z}_{15}$. For each subgroup, list out all the left cosets. Express your answers in set notation (e.g. $\{1, 5, 9, 11\}$).
8. Let $G$ be a group, and suppose $H$ is a subset. Consider the following theorem. If $H$ is non-empty and for any pair of elements $a, b \in H$ we know that $ab^{-1} \in H$, then $H$ is a subgroup. Fill in the blanks in this proof.

There is some element $a \in H$, because _____________

Therefore, $a, _____________ \in H$, so _____________ = _____________ must be in $H$.

Similarly, $a, _____________ \in H$, so _____________ = $a^{-1}$ must be in $H$.

It follows that for any $a \in H$, $a^{-1} \in H$.

Suppose $a, b \in H$. We know that _____________ \in H.

Therefore, $a, _____________ \in H$, and so _____________ = $ab \in H$.

We have shown that the operation is closed in $H$, the identity is in $H$, and for each element of $H$, its inverse is also in $H$.

Therefore, $H$ must be a _____________ of $G$.