

A.  $(p \vee t) \rightarrow (p \rightarrow \neg t)$ ,  $(p \rightarrow \neg t) \rightarrow (t \rightarrow q)$ ,  $p \vee t \Rightarrow p \vee q$

B.  $(q \wedge s) \rightarrow r$ ,  $q \vee p$ ,  $s \Rightarrow r \vee p$

C.  $q \leftrightarrow r$ ,  $q \rightarrow (r \rightarrow p)$ ,  $\neg p \Rightarrow \neg r$

D.  $s \rightarrow (\neg p \rightarrow q)$ ,  $p \rightarrow \neg s$ ,  $\neg q \Rightarrow \neg s$

A.  $(p \vee t) \rightarrow (p \rightarrow \neg t)$ ,  $(p \rightarrow \neg t) \rightarrow (t \rightarrow q)$ ,  $p \vee t \Rightarrow p \vee q$

1.  $(p \vee t) \rightarrow (p \rightarrow \neg t)$       Given.
2.  $(p \rightarrow \neg t) \rightarrow (t \rightarrow q)$       Given.
3.  $(p \vee t) \rightarrow (t \rightarrow q)$       Chain rule on 1 and 2.
4.  $p \vee t$       Given.
5.  $t \rightarrow q$       Modus Ponens on 3 and 4.
6.  $\neg p \rightarrow t$       Conditional Equivalence on 4.
7.  $\neg p \rightarrow q$       Chain rule on 5 and 6.
8.  $p \vee q$       Conditional Equivalence on 7.

B.  $(q \wedge s) \rightarrow r, q \vee p, s \Rightarrow r \vee p$

1.  $(q \wedge s) \rightarrow r$                       Given.
2.  $\neg(q \wedge s) \vee r$                       Conditional Equivalence on 1.
3.  $\neg q \vee \neg s \vee r$                       DeMorgen's law on 2.
4.  $\neg q \vee r \vee \neg s$                       Commutative law on 3.
5.  $s$     Given.
6.  $\neg q \vee r$                               Disjunctive Simplification on 4 and 5.
7.  $q \rightarrow r$                               Conditional Equivalence on 6.
8.  $q \vee p$                                 Given.
9.  $\neg p \rightarrow q$                           Given.8, after using commutativity on 8.
10.  $\neg p \rightarrow r$                           Chain rule on 9 and 7.
11.  $r \vee p$                                 Conditional Equivalence on 10, then using commutativity.

C.  $q \leftrightarrow r, q \rightarrow (r \rightarrow p), \neg p \Rightarrow \neg r$

1.  $q \rightarrow (r \rightarrow p)$  Given.
2.  $\neg q \vee (r \rightarrow p)$  Conditional Equivalence on 1.
3.  $\neg q \vee (\neg r \vee p)$  Conditional Equivalence on 2.
4.  $(\neg r \vee \neg q) \vee p$  Associativity and commutativity on 3.
5.  $\neg p$  Given.
6.  $\neg r \vee \neg q$  Disjunctive Simplification on 4 and 5.
7.  $q \leftrightarrow r$  Given.
8.  $q \rightarrow r \wedge r \rightarrow q$  Biconditional Equivalence on 7.
9.  $r \rightarrow q$  Conjunctive Simplification.
10.  $\neg r \vee q$  Conditional Equivalence on 9.
11.  $(\neg r \vee q) \wedge (\neg r \vee \neg q)$  on 6 and 10.
12.  $\neg r \vee (q \wedge \neg q)$  Distribution on 11.
13.  $\neg r \vee 0.$  Negation on 12.
14.  $\neg r.$  Null law on 13.