

How many ways can you line up the letter A?

One way, A. How exciting.

How many ways can you line up the letters A and B?

Two. AB and BA.

How many ways can you line up the letters A, B, and C?

Six. Take each of AB and BA from the question for two letters. You can put the C in the back, middle or front of each. ABC, ACB, and CAB for AB and BAC, BCA, and CBA for BA.

What about ABCD? How many ways are there to arrange these 4 letters? There are 6 ways to line up 3 letters. For each one of these, I can then put the D in the back, next to the back, second spot, or up front. So the total number is 4 (because there's 4 places we can put D) times the number of ways of lining up 3 items.

This logic worked for the beginning ones also.

1 letter, one rearrangement.

2 letters has $2 * 1$ rearrangements.

3 letters has $3 * (2 * 1)$ rearrangements.

4 letters has $4 * (3 * 2 * 1)$ rearrangements.

This pattern keeps going.

It happens often enough in math that we've built a function for it. We call it factorial and use ! as the symbol for it.

$$1! = 1$$

$$2! = 2 * 1$$

$$3! = 3 * 2 * 1$$

$$4! = 4 * 3 * 2 * 1$$

And in general: $n! = n * (n-1) * (n-2) * \dots * 3 * 2 * 1$

Using a calculator you should be able to find that $8! = 40320$.

How is this relevant to weighted voting? It's a way of checking if you have written all sequential coalitions.

How many sequential coalitions are there with 8 players? It's the number of ways of lining up A, B, C, D, E, F, G, and H. That would be $8!$ which we have above as 40320. That would be a really mean problem, huh?