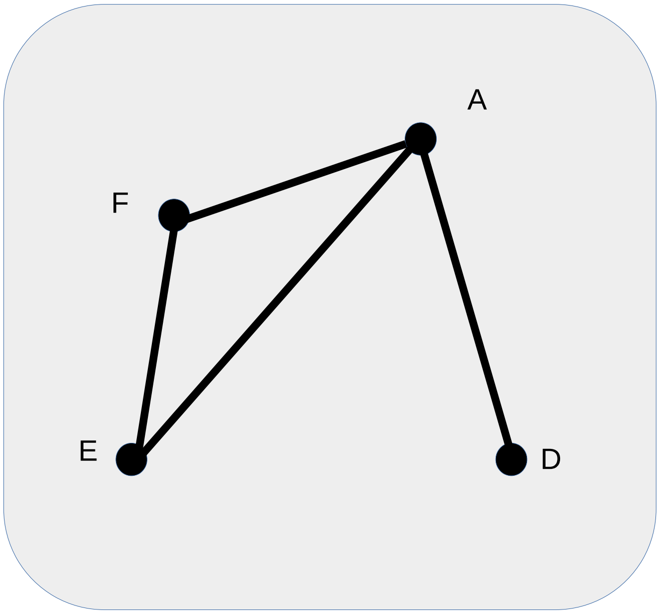
Graph Theory Terms And Concepts (some are not in the book)

First—not all Graph Theorists necessarily agree on all the definitions I'm about to list. In this class, use my definition above anything you find anywhere else. I'm also not planning on discussing the set theory that underlies graph theory. We can do quite a bit with graphs without getting into the hardcore mathematics of it all. I don't know what other professors tell you about Wikipedia, but the math on Wikipedia is usually spot-on if maybe a little tough for non-mathematicians. When you're done reading this, you should check out the Wiki on "Graph Theory".

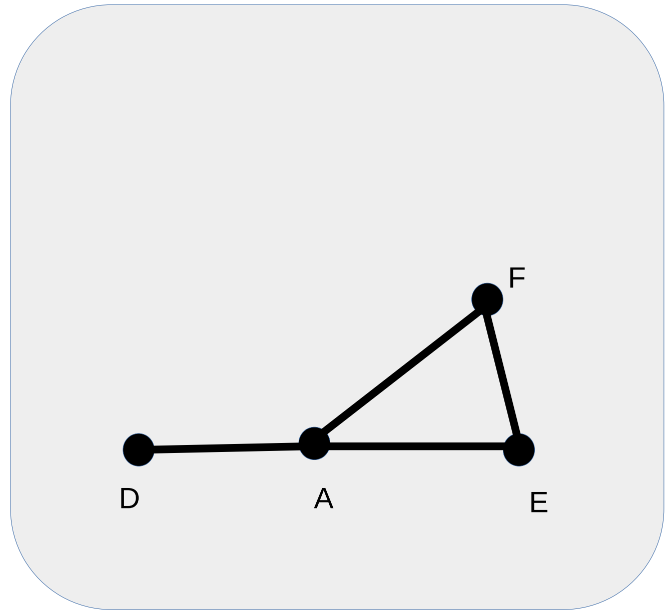
A **graph** consists of a set of "dots" and a set of "lines".

The dots are called **vertices** (singular vertex) and the lines are called **edges** (singular edix. Just kidding. It's edge, but you knew that.)

When you start out with graphs, pictures are usually easier to deal with.



Graph G_1

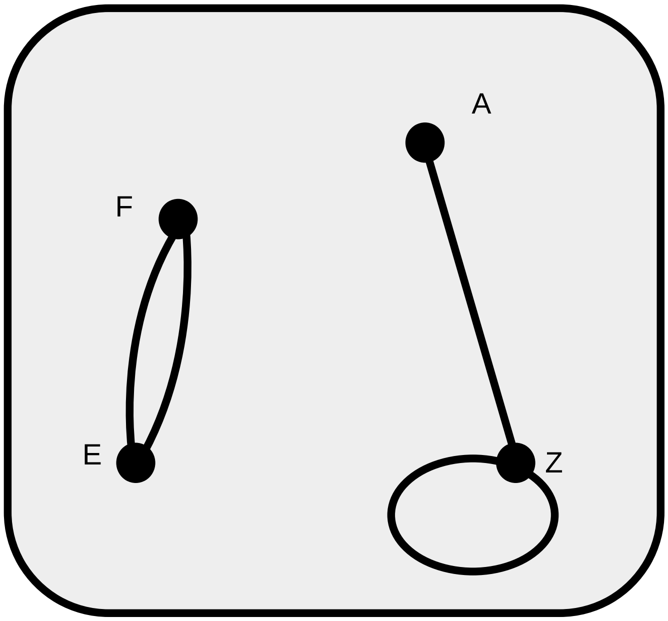


Graph G_2

These two graphs are the same; graphs don't have the same type of geometry that you'd see in a the graph of a function or a standard geometry class. The only information a graph contains is the vertices and how they are connected with the edges.

Typically we name the vertices with single letters for simplicity. The edges are named from the vertices they connect. So the edge that connects vertex F and vertex E is simply named EF or FE. The order doesn't matter in a regular graph.

We say that two vertices are **adjacent** if there is an edge connecting them. Likewise, **adjacent edges** share a common vertex. In graph G_1, F and E are adjacent vertices. F and D are not. AD and FA are adjacent edges whereas FE and AD are not.



Graph G_3

Graph G_3 is indeed a graph that includes all 4 vertices A, E. F and Z. Unlike Graph G_1, this one is **disconnected**. Logically, graph G_1 is considered **connected**. Notice that there are two copies of edge EF. We call this **multiple edges**. Also notice that there is an edge that connects vertex Z with itself. The edge's name would be ZZ, and we call this type of edge a **loop**.

Most graph theorists would call a graph without any loops or multiple edges a "simple graph."

Every edge has two "ends". The **degree of a vertex** is the number of these ends that connect to that vertex. So if you go back to the first graph, G_1, you can see that:

vertex D has degree 1.
vertex A has degree 3.
vertex E has degree 2.
vertex F has degree 2.

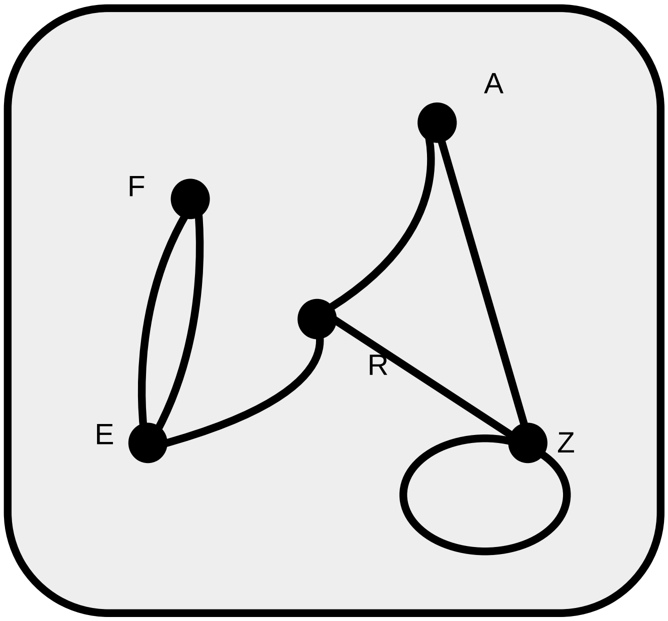
In graph G3, we have:

vertex Z has degree 3.
vertex A has degree 1.
vertex E has degree 2.
vertex F has degree 2.

Soon we will care about the parity of the degree of the vertices. We say that a vertex that has degree that is an even number a **vertex of even degree**, or more compactly as an **even vertex**. Similarly, if the degree is odd, it is an **odd vertex**.

(This is different than the book.) A **path** is a sequence of adjacent vertices without using any edge more than once. The length is the number of edges used.

If it starts and ends at the same vertex it is called a **circuit**.



Graph TU. (Tortured Undies)

Examples: ARZ is a path of length 2.
ARZA is a circuit of length 3.
ARZF is not a path because Z and F are not adjacent.
AZ is a path of length 1.
ZZ is a circuit of length 1.
ARA is not a path nor a circuit because it reuses an edge.
But FEF is a perfectly fine circuit.

I've already mentioned connected/disconnected. Now I can define it for you: if for every different pair of vertices there is a path that starts at one and ends at the other, we say the graph is **connected**. A graph that is not connected is **disconnected**.

An edge that, when removed from the graph, changes the graph from connected to disconnected is called a **bridge**. It's an edge that is critical for holding the graph together. There is exactly one bridge in graph TU. It's edge ER.

Before you read the book, pages 117-132 (Stop before Hamilton circuits), keep in mind a couple of things:

1. Don't worry at all about Dijkstra's algorithm yet. So you can skip most of pages 122-127. I teach that a totally different way. I find the book's method not terribly clear.

Euler's first theorem (not in the book) The sum of all degrees of all vertices is even. In fact if there are n edges, the sum is 2n.

Why? I think the easiest way to see it is by re-building the graph in your mind. Imagine the graph with zero edges. Every time you add an edge it has two ends. So the sum of all degrees of all vertices is increased by 2 every time you add a new edge.

Euler's second and third theorems The book has them listing in a box on the bottom of page 128. They're both a little bit wrong. They should say:

A **connected** graph will contain an Euler path if it contains at most two vertices of odd degree.

A **connected** graph will contain an Euler circuit if all vertices have even degree.