

Voting Theory – Framework

- All elections are assumed to use preference ballots.
- And we can summarize an entire election using a preference schedule.

Voting Theory – Framework

Suppose an election resulted in the table to the right.

	6	4	3	3	1
A	A	C	B	B	A
C	C	D	D	C	B
D	D	B	C	D	C
B	B	A	A	A	D

For most of the rest of the chapter, we will be doing two things.

- Discussing how we choose the winner of the election. We'll call these **methods**.
- Deciding “criticisms” of any method being used. We will call these **criteria**.

Voting Theory – Framework

Let's start by introducing the “normal” way of voting. First, we need two terms.

6	4	3	3	1
A	C	B	B	A
C	D	D	C	B
D	B	C	D	C
B	A	A	A	D

A **plurality** is the largest subgroup of a population that's divided into subgroups. In the context of voting, one might say “Candidate X received a plurality of votes.” What that means is that X received the most 1st place votes of all the candidates.

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B	A	A	A	D

A **majority** of anything means more than half. Strictly more than half. Again, if you were to say that X received a majority of votes, the implication is first place votes.

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Is it possible to have a majority of votes, but not a plurality?

(Think before going on.)

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Is it possible to have a majority of votes, but not a plurality?

Suppose A has a majority, but not a plurality. Someone else has a plurality. Call them candidate B. Since A has a majority, it has at least 50% of the votes. B has a plurality, so B must have more votes than A. How many voters does A and B have together? They are both more than 50%, so greater than 100%. That's silly. It's not possible. What that means is:

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Having a majority of votes automatically means having a plurality of votes.

6	4	3	3	1
A	C	B	B	A
C	D	D	C	B
D	B	C	D	C
B	A	A	A	D

It's not true backwards. Look at the election to the right. A has 7 votes, C has 4 votes, B has 6 votes and D has no votes. (Just looking at 1st place votes here.)

There are a total of 17 voters. Half is 8.5, so a majority is at least 9 votes. A has more than anyone else, so A is a plurality candidate. But A does not have a majority.

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The “normal” way of voting is called **the plurality method**. The candidate with a plurality of 1st place votes wins.

6	4	3	3	1
A	C	B	B	A
C	D	D	C	B
D	B	C	D	C
B	A	A	A	D

- In a standard election, we only collect 1st place votes. “Most votes wins” is how we do it.

Is the plurality method any good? To answer that, we need a set of criteria to judge the method.

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Under plurality, we see that A wins the election to the right.

	6	4	3	3	1
A	A	C	B	B	A
C	C	D	D	C	B
D	D	B	C	D	C
B	B	A	A	A	D

The first criterion (plural is criteria) for this class is **the majority criterion**. The majority criterion states that “in an election, if a candidate gets a majority (of 1st place votes), then that candidate should be the winner.”

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majority criterion. In an election, if a candidate gets a majority (of 1st place votes), then that candidate should be the winner.

When we “judge” an election method with a criterion we are asking a fairly complicated question. When we say that the <Bird> method satisfies the <Fish> criterion what we actually mean is this: no matter how many candidates are running, no matter how many voters there are, no matter how they vote, the <Bird> method never violates the <Fish> criterion. Let me make up a silly criterion to illustrate this.

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Define the Drool criterion as this: “The candidate with the shortest name should win the election. In the event of a tie, the candidate that comes first alphabetically should win.”

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Using the plurality method on the election to the right, we already said A wins. As it turns out, according to the Drool criterion, A should win because all the candidates have equal length names, and A is alphabetically first.

	6	4	3	3	1
A	A	C	B	B	A
C	C	D	D	C	B
D	D	B	C	D	C
B	B	A	A	A	D

In this election, the plurality method does not violate the Drool criterion.

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But consider an election with 2 candidates X and Y. 100 people vote, and all of them prefer Y over X. Plurality states that Y wins. But the Drool criterion says that X should win.

Because there is at least one election situation that the plurality method fails the Drool criterion, we say that “Plurality does not satisfy the Drool criterion.”

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Under plurality, A wins the election to the right.

	6	4	3	3	1
A	A	C	B	B	A
C	C	D	D	C	B
D	D	B	C	D	C
B	B	A	A	A	D

What does the majority criterion think about this? Actually, it doesn't care! Nobody got a majority, so the majority criterion is “vacuously” satisfied in this specific election.

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So what do you think? Does the plurality method satisfy the majority criterion? Can you come up with a coherent logical argument to justify your position?

Post your thoughts in the discussion board on D2L.