Voting Theory Part II

Before I get into new material, I want to show you how a mathematician would answer “does the plurality method satisfy the majority criterion?”

First, note that it’s impossible to have more than one majority candidate. If there were two or more majority candidates they would have a total of more than 100% of the votes.

The answer is yes. Suppose there is an election that does not have a majority candidate. Then we certainly know the majority criterion is not violated. And if the election does have a majority candidate they are the plurality candidate, for if they weren’t there would be more than one majority candidate. Thus any majority candidate always wins under the plurality method.

Voting Theory Part II

- All elections are assumed to use preference ballots.
- And we can summarize an entire election using a preference schedule.
- Voting methods are how we choose the winner of an election.
- Voting criteria are “standards” for critiquing voting methods. Ideally we choose a voting method that always always always satisfies any criteria we think are fair.

Voting Theory Part II

- Methods so far: plurality. (Most 1st place votes wins.)
- Criteria so far: majority criterion. (If candidate gets majority of 1st place votes, they should win.)

I'm going to introduce a new criterion: the Condorcet criterion. But first, we need some background.

Voting Theory Part II

Before we move on, here's an important idea captured by a preference schedule under the assumption of transitivity.

6	4	3	3	1
A	C	B	B	A
C	D	D	C	B
D	B	C	D	C
B	A	A	A	D

Suppose the results of an election are captured in the preference schedule to the right. What happens if some candidate is removed from the election?

Voting Theory Part II

Let's say that candidate C is disqualified. We don't have to redo the whole election because we already know how the voters feel. If I rank the candidates as BCDA (B is 1st, A is 4th) and C is eliminated then my ballot automatically becomes BDA.

6	4	3	3	1
A	C	B	B	A
C	D	D	C	B
D	B	C	D	C
B	A	A	A	D

In a standard election where you vote for the top candidate only, this DOES NOT work. You need a new election if you actually care about the will of the people.

Voting Theory Part II

So candidate C is disqualified. Then the preference schedule above becomes the one below. (Basically, remove C and take up the slack.)

6	4	3	3	1
A	C	B	B	A
C	D	D	C	B
D	B	C	D	C
B	A	A	A	D

(Technically the 3rd and 4th column become the same thing, and we should combine them into one column of 6 voters.)

6	4	3	3	1
A	D	B	B	A
D	B	D	D	B
B	A	A	A	D

Voting Theory Part II

A **head-to-head comparison** of two candidates is whoever wins the most 1st place when all the other candidates are eliminated.

Voting Theory Part II

Suppose we have a head-to-head comparison between candidates B and D in the election to the right. It's not all that complicated. Remove everyone else (A and C) and count 1st place votes.

6	4	3	3	1
A	C	B	B	A
C	D	D	C	B
D	B	C	D	C
B	A	A	A	D

6	4	3	3	1
D	D	B	B	B
B	B	D	D	D

In this case D gets 10 and B gets 7. D wins.

Voting Theory Part II

One way we like to sort out a set of teams is by letting every team play every team. It is hard to argue against a team that wins every single game, right? Let's put this on candidates.

If a candidate wins every head to head comparison against every other candidate, we call that candidate a **Condorcet candidate**. That sentence candidates.

If a player wins every H2H against all other players, they are the Condorcet candidate.

Voting Theory Part II

Consider the election to the right. Is there a Condorcet candidate?

4	6	3	2
A	B	C	C
D	D	D	A
C	C	A	D
B	A	B	B

Voting Theory Part II

Consider the election to the right. Is there a Condorcet candidate?

(We need to check some head to head contests. Remember, remove everyone except the pair in question. Who wins the most 1st place votes?)

5	6	3	2
A	B	C	C
D	D	D	A
C	C	A	D
B	A	B	B

- A vs B. A wins 10 to 6. B can't be a Condorcet Cand.
- A vs C. C wins 11 to 5. A isn't either.
- C vs D. D wins 11 to 5. C is out.
- D vs A. D wins 9 to 7.
- D vs B. D wins 10 to 6.

D beat A, B, and C in H2H contests. That's everyone. So, D is a Condorcet Candidate.

Voting Theory Part II

Again, if a candidate wins every head to head comparison against every other candidate, we call that candidate a Condorcet Candidate.

An election does not always have a Condorcet Candidate. (Just like an election does not always have a Majority Candidate.)

Voting Theory Part II

Time for our second criterion. The **Condorcet criterion** is that any time there is a Condorcet candidate, that candidate should win the election.

Voting Theory Part II

Does the plurality method satisfy the Condorcet criterion?

Again, this question does not mean “in this one specific election it does.” Nor does it mean “most of the time it does.”

If there is a single possible election with any number of candidates and voters for which the plurality method picks a winner that isn’t a Condorcet candidate then the answer is “no”.

Voting Theory Part II

Check it out. In the last election, we determined that D is a Condorcet Candidate. Who wins this election under plurality?

5	6	3	2
A	B	C	C
D	D	D	A
C	C	A	D
B	A	B	B

B wins under plurality. D received zero first place votes!!

Plurality violates the Condorcet criterion.

Look at that election as if you actually cared about who wins. B is the winner under our current system, plurality. But you've really got to ask yourself, is this really the will of the people? Yeah, yeah, B got 6 out of 16 first place votes. BUT everyone else placed B in last place! B received a majority of last place votes!

Sign on the 10 other peoples lawn ==> Anyone but B!

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